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Modelling and constructing membership function for uncertain portfolio parameters: A credibilistic framework

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ABSTRACT

In this paper we attempt to automate the process of fitting the uncertain parameters of a multi-objective portfolio selection problem by generating L-R fuzzy numbers that belong to power reference function family. Such an approach is advantageous when the fuzzy parameters of the portfolio are best represented as general functional forms. We also propose four new portfolio selection models in a multi-criteria credibilistic framework. The key financial criteria considered are return, illiquidity (antagonistic to liquidity) and risk. In the absence of joint possibility distribution of these parameters, the return and illiquidity of the entire portfolio are considered as historical data set instead of return and illiquidity of the individual assets. In the process of fitting the most appropriate L-R fuzzy number, the vagueness within the information and deviation of the L-R fuzzy number from historical data is measured using entropy and cross-entropy respectively. These principles are embedded into the modelling process of proposed portfolio selection problems. One of the key contribution of this study is that we propose and design a sub-algorithm namely "Entropy-Cross Entropy (ECE) Algorithm" that is appended within an "MIBEX-SM" genetic algorithm and is used to solve the proposed portfolio optimization problems. This proposed solution methodology results in an automated system that is intelligent enough to extract information required for fitting of L-R fuzzy number from the historical data and does not need any human intervention in terms of stating the parameters of the problem. We also conduct an empirical study to demonstrate the impact of the solution approach and applicability of the proposed models in practical applications of portfolio selection. For this purpose we collected historical data from National Stock Exchange (NSE), Mumbai, India. The data for a period of 2008–2013 is first used to train the models. Then the data sets of one-year period of 2013 – 2014 and two-year period of 2013 – 2015 are used to test the performance of the models. We introduce a decision support system strategy of comparing the performance of models by proposing a modified version of Sharpe ratio in fuzzy context, named as "Credibilistic Sharpe Ratio (CrSR)".

1. Introduction

Since the advent of pioneering work by Markowitz (1952) in early 1950s, the field of portfolio selection has grown manifold. Owing to various dimensions of investment market, the mean-variance model as proposed by Markowitz has been extended to a multi-criteria framework by many researchers (Anagnostopoulos & Mamanis, 2010; Ehrgott, Klamroth, & Schwehm, 2004; Sharma & Mehra, 2016; Xidonas, Mavrotas, Krintas, Psarras, & Zopounidis, 2012; Zopounidis & Doumpos, 2013). One of the key consideration other than modelling risk and return while formulating these models is, liquidity of the portfolio (Bogdan, Bareša, & Ivanović, 2012; Cannon & Cole, 2011; Gupta, Inuiguchi, Mehlawat, & Mittal, 2013a; Haliassos & Michaelides, 2003; Von Wyss, 2004). While making an investment decision in the financial markets, investors often face scenarios that are described using linguistic expressions such as "high risk", "low returns" and "low liquidity". Such vagueness and ambiguity in the investment market has been modelled using fuzzy set theory (Zadeh, 1965).

A rich literature is available where researchers have used possibility measure to model portfolio selection problems (Dubois & Prade, 1985; Tanaka & Watada, 1988; Wang & Zhu, 2002; Wang & Klir, 2013). With Liu and Liu (2002) proposing a self-dual credibility measure to overcome the limitations of possibility measure, a paradigm shift took place while modelling portfolio selection problems. Since then, credibility measure has become a

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popular measure for the development of fuzzy portfolio selection models (Huang, 2006; 2008a; Li, Li, Qin, & Cheng, 2014; Vercher & Bermúdez, 2015). A major aspect of such a fuzzy modelling is an appropriate selection of membership function that will best fit the historical data. The literature is full of usage of conventional membership functions such as linear, trapezoidal, triangular and S-shaped (Gupta, Mittal, & Mehlawat, 2013b; Jalota & Thakur, 2016; Li, Zhang, Wong, & Qin, 2009; Mashayekhi & Omrani, 2016; Qin, Li, & Ji, 2009; Saborido, Ruiz, Bermúdez, Vercher, & Luque, 2016) for this purpose. In an endeavor to look for a better fit of the fuzzy representation of historical data, one needs a more flexible membership function as generated by L-R fuzzy numbers (Dubois & Prade, 1978; Saborido et al., 2016; Vercher & Bermudez, 2013; Vercher & Bermúdez, 2015). The appropriateness of fit can be studied as an optimization problem wherein, an entropy function is used to measure the vagueness within the information and a cross entropy function measures the deviation of the fit of fuzzy membership function from the actual data (Cheng & Chen, 1997; Hasuike, Katagiri, Tsubaki, & Tsuda, 2012; Li, Qin, & Kar, 2010; Nieradka & Butkiewicz, 2007).

1.1. Research issues and need for the present work

The studies reviewed so far are based on fitting of membership function to the uncertain parameters of the portfolio using the conventional membership functions that are triangular or trapezoidal in nature. There exists only few studies (Saborido et al., 2016; Vercher & Bermudez, 2013; Vercher & Bermúdez, 2015) that are based on fitting of L-R fuzzy numbers to the uncertain portfolio parameters. The limitation of these studies is that, one has to feed the parameters required for such a fitting manually. This brings in a lot of subjectivity while fitting such L-R fuzzy numbers. Thus, there is a need for an expert system that can do away with such a subjectivity and extracts information embedded within the data set to find the most appropriate fit.

Another issue that needs to be addressed is the use of possibility measure to model the uncertain parameters of portfolio optimization problems. Possibility measure fails self-duality property and a more realistic, self-dual credibility measure can be a good replacement for it. The studies in literature are usually based on fitting of L-R fuzzy number in possibilistic framework. To the best of our knowledge there is only one study (Vercher & Bermúdez, 2015) that sets up credibilistic framework for fitting L-R fuzzy number to uncertain portfolio parameters.

Again, to the best of our knowledge there is no study in literature that compares the formulated portfolio optimization models for their performance in a credibility based fuzzy environment. Moreover, in a similar environment there does not seem to exist a study that trains the proposed models and then uses a futuristic data set to test their performances. However, for a decision maker it is important that there exist a robust system that helps him/her pick the most reliable model to make investment related decisions.

These issues were our motivating factors to come up with this study.

1.2. Key differences between closely related studies and the work presented in this paper

The key differences between closely related studies (Bhattacharyya, 2013; Nguyen, Gordon-Brown, Khosravi, Creighton, & Nahavandi, 2015; Saborido et al., 2016; Vercher & Bermudez, 2013; Vercher & Bermúdez, 2015) from literature and our study are as follows:

(i) In the studies available in literature that are related to fitting of L-R fuzzy number to uncertain portfolio parameters (Saborido et al., 2016; Vercher & Bermudez, 2013; Vercher & Bermúdez, 2015), the parameters of fitting are subjected to the opinion of the decision maker or stock experts. Such a decision making becomes very subjective in nature. In this study we design an automated system of fitting L-R fuzzy numbers. An intelligent system is proposed that can extract information from the data set and minimize the deviation of the fitted L-R membership function from the actual historical data involved in portfolio formation.

(ii) In literature, L-R fitting of fuzzy number has been usually taken up within a possibility based framework (Saborido et al., 2016; Vercher & Bermudez, 2013). Though in Vercher and Bermúdez (2015), a portfolio optimization model is proposed in credibility based framework but the L-R fitting is done using experts opinion on parameters of the L-R fuzzy numbers. In this paper we also propose portfolio optimization models involving L-R fitting of fuzzy numbers in credibilistic framework. The main difference between our work and the work proposed by Vercher and Bermúdez (2015) is that, in order to find the most appropriate L-R fuzzy number that will fit the historical data we propose credibility based optimization objectives of entropy and cross entropy. These objectives measure the vagueness within the information and deviation of the L-R power fuzzy number from the historical data set.

(iii) The proposed models involving L-R fitting in literature (Saborido et al., 2016; Vercher & Bermudez, 2013; Vercher & Bermúdez, 2015) are solved using multi-objective evolutionary algorithm. In this study we propose a new algorithm namely ECE algorithm embedded within an "MIBEX-SM" genetic algorithm to solve the proposed optimization models. This algorithm is a generic algorithm and results in automatic fitting of L-R fuzzy number. Another key highlight of this algorithm is that it always ensures that the new solutions created after the process of crossover and mutation are within the bounds and retain their feasibility.

(iv) Usually in literature, a single portfolio optimization model involving L-R fitting, has been proposed (Saborido et al., 2016; Vercher & Bermudez, 2013; Vercher & Bermúdez, 2015). In this work, we propose four new portfolio optimization models in credibilistic framework of L-R fitting of fuzzy numbers, with one of the main objectives being optimization of entropy and cross entropy functions. Next, we use historical data from NSE, Mumbai, India for training the proposed models and then testing them for their performance on the data for some later period. To the best of our knowledge, whenever more than one model have been proposed in literature, the models have not been compared with respect to each other using a futuristic data set. Such a strategy is a key highlight of any decision support system and adds to the novelty of our solution approach.

(v) Owing to the credibilistic fuzzy context of portfolio selection problems being handled, we propose a modified version of Sharpe ratio, namely CrSR to compare and pick the most robust of all the models. When compared to existing literature, Bhattacharyya (2013) proposed a possibilistic Sharpe ratio and Nguyen et al. (2015) proposed a modified Sharpe ratio in fuzzy random environment. To the best of our knowledge, such a modification of Sharpe ratio has not been done in credibilistic environment.

1.3. Focus of the present study

In this paper, we generate L-R fuzzy numbers that belong to the family of power reference functions, to model the uncertain parameters of a multi objective portfolio selection problem in