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Alliance-based evidential reasoning approach with unknown evidence weights

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ABSTRACT

In the evidential reasoning approach of decision theory, different evidence weights can generate different combined results. Consequently, evidence weights can significantly influence solutions. In terms of the "psychology of economic man," decision-makers may tend to seek similar pieces of evidence to support their own evidence and thereby form alliances. In this paper, we extend the concept of evidential reasoning (ER) to evidential reasoning based on alliances (ERBA) to obtain the weights of evidence. In the main concept of ERBA, pieces of evidence that are easy for decision-makers to negotiate are classified in the same group or "alliance." On the other hand, if the pieces of evidence are not easy to negotiate, they are classified in different alliances. In this study, two negotiation optimization models were developed to provide relative importance weights based on intra- and inter-alliance evidence features. The proposed models enable weighted evidence to be combined using the ER rule. Experimental results showed that the proposed approach is rational and effective.

1. Introduction

Evidence theory (Dempster, 1967; Shafer, 1976) was originally developed by Arthur P. Dempster and later extended and refined by Glenn Shafer. It is thus sometimes referred to as the Dempster–Shafer (DS) theory of evidence. DS theory is a powerful and flexible mathematical tool for addressing imprecise and uncertain information. Hence, it has been employed in areas such as expert systems (Beynon, Cosker, & Marshall, 2001), uncertainty reasoning (Jones, Lowe, & Harrison, 2002), pattern classification (Denoeux & Masson, 2004), fault diagnosis and detection (Fan & Zuo, 2006), information fusion (Telmoudi & Chakhar, 2004), multiple attribute decision analysis (Xu, Yang, & Wang, 2006), image processing (Huber, 2001), risk analysis (Deng, Sadiq, Jiang, & Tesfamariam, 2011), e-commerce security (Zhang, Deng, Wei, & Deng, 2012), financial applications (Dymova, Sevastianov, & Kaczmarek, 2012; Dymova, Sevastjanov, & Kaczmarek, 2016), and water distribution systems (Bazargan-Lari, 2014).

Meanwhile, decision theory, or the theory of choice, is the study of the reasoning behind a decision-maker's choice. It is used to solve problems involving selection from a finite number of choices. Decision theory literature has an extensive history. However, many studies published in this area, such as works on the analytic hierarchy process (AHP), assume complete information in the decision-making process. That is, the methods assume that decision-makers are fully aware of their specific preferences. In cases in which data for assessing alternatives against criteria are partially or completely unavailable, or the decision-maker's knowledge of an alternative evaluation is insufficient, the decision-makers are more likely to use uncertain assessment information.

DS theory, on the other hand, is well suited to handling uncertainty. It is particularly useful for dealing with uncertain subjective judgments when multiple pieces of evidence must be simultaneously considered. An evidential reasoning (ER) approach based on both decision theory and DS theory was thus proposed by Yang and Singh (1994). In the past two decades, the original ER approach has been extensively developed to solve multi-attribute decision making (MADM) problems with uncertainties, including fuzzy evaluation grades, interval evaluation grades, fuzzy belief structures, interval belief degrees, dynamic belief degrees, partially ordered preferences under uncertainty, and unknown attribute weights in various values and preference judgments (Fu & Chin, 2014; Guo, Yang, Chin, Wang, & Liu, 2009; Hu, Si, & Yang, 2010; Huynh, Nakamori, Ho, & Murai, 2006; Wang, Yang, Xu, & Chin, 2006; Yang, 2001; Yang & Xu, 2002a; Yang & Xu, 2002b; Yang, Wang, Xu, & Chin, 2006).

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Furthermore, the ER approach and its extensions have been widely applied to MADM problems in business performance assessment (Yang, Dale, & Siow, 2001), pre-qualification of construction contractors (Sönmez, Yang, Graham, & Holt, 2002), environmental impact assessment (Wang, Yang, & Xu, 2006), organizational self-assessment (Siow, Yang, & Dale, 2001), safety analysis (Liu, Yang, Wang, SII, & Wang, 2004; Wang & Yang, 2001), bridge condition assessment (Wang & Elhag, 2007), behavior prediction (Zhou, Hu, Xu, Yang, & Zhou, 2010), fault prediction (Si, Hu, Yang, & Zhang, 2011), risk analysis (Tang, Yang, Chin, Wong, & Liu, 2011), job offering (Mahmud, Rahman, & Hossain, 2013), software selection (Chin & Fu, 2014), and group decision analysis (Fu, Huhns, & Yang, 2014).

In the above ER approaches, the assessment information for each criterion is regarded as a piece of evidence; the criterion weight provides the evidence weight. The residual support remains uncommitted because the evidence weight is assigned to any singleton proposition and the universal set proposition, which contains all elements of a proposition. This specific assignment can differentiate between ignorance and residual support, while reflecting the relative importance of other evidence. As Xu (2009) contended, this specificity enables the ER approach to solve counterintuitive problems in which conflicting pieces of evidence are combined using Dempster's rule.

However, most existing ER approaches have made significant advancements in solving MADM problems with different decision scenarios based on the original ER algorithm. In this algorithm, it is assumed that local ignorance exists in none of the evidence. While this assumption is reasonable when solving MADM problems, it is difficult to apply to other domains.

To expand the range of ER applications, Yang and Xu (2013) relinquished this assumption and generalized the ER algorithm into a new weighted ER rule that accounts for both global and local ignorance. In addition, they further expanded the ER rule to combine multiple pieces of evidence according to their weights and degrees of reliability. These advancements have considerably enriched ER theory. It is said that the importance of a piece of evidence reflects a decision-maker's preferences over the evidence. The importance is thus subjective; it depends on the agent making the judgment when using the evidence.

Nevertheless, none of the above ER approaches explain how to determine the relative importance of the evidence weight. Moreover, the individual behavior of the decision-maker is not considered. Because the results are interest-driven, it is impossible for the decision-maker to have an unbiased, isolated perspective regarding the evidence. In certain decision-making situations, the determining agent seeks similar pieces of evidence to support their own evidence, thereby forming an alliance of evidence. We therefore propose the "evidential reasoning" approach (ERBA), which is based on alliances.

The remainder of this paper is organized as follows. Section 2 introduces the relevant concepts of DS theory, evidential reasoning, and the pignistic probability distance. Section 3 describes the importance of evidence weight. Section 4 details the development of the ERBA approach, and Section 5 analyzes the rationality of the proposed approach. Section 6 concludes the paper.

2. Preliminaries

In this section, we introduce some prior knowledge regarding DS theory, the evidential reasoning algorithm, and the pignistic probability distance, which is used as the basis for later discussions.

2.1. DS theory

DS theory is viewed as a generalization of probability theory that can handle multiple possible propositions, e.g., sets of propositions. Let $\Theta = \{H_1, \cdots, H_N\}$ be a collectively exhaustive and mutually exclusive set of propositions. It is called the frame of discernment. Three important functions exist in DS theory: the *basic probability assignment* function (bpa or m), *belief* function (*Bel*), and *plausibility* function (*Pl*). These functions are defined below.

Definition 1. (Dempster, 1967) **.** A basic belief assignment (bba) (also called a belief structure) is a function, $m : 2^\Theta \to [0, 1]$ (called the *basic probability assignment* (bpa) in Shafer's original definition), which satisfies

$$\begin{cases} m(\phi) = 0, \\ \sum\limits_{A \subseteq \Theta} m(A) = 1 (0 \le m(A) \le 1), \end{cases}$$

where ϕ is an empty set, A is a subset of Θ, and 2^Θ is the power set of Θ, which consists of all the subsets of Θ, i.e., $2^\Theta = \{\phi, \{H_1\}, \cdots, \{H_N\}, \{H_1, H_2\}, \cdots, \{H_1, H_N\}, \cdots, \Theta\}$. The function $m(A)$ is the bpa of A. It measures the belief explicitly assigned to A and represents how strongly the evidence supports A. Each subset A is called a focal element of m. All related focal elements are collectively called the body of evidence.

Definition 2. (Dempster, 1967) **.** Both the belief function and plausibility function can be derived from the basic probability assignment, $m(A)$, as

$$Bel(A) = \sum\nolimits_{B \subseteq A} m(B) \text{ and } Pl(A) = \sum\nolimits_{B \cap A \neq \phi} m(B)$$

In this definition, $Bel(A)$ represents the sum of the basic probability masses assigned to A, whereas $Pl(A)$ represents the sum of possible basic probability masses that could be assigned to A.

The core concept of DS theory is Dempster's rule, by which pieces of evidence from different sources are combined or aggregated. This rule assumes that information sources are independent. It thus uses the so-called orthogonal sum to combine multiple belief structures as $m_1 \oplus m_2 \cdots \oplus m_n$, where $\oplus$ represents the combination operator. With multiple bbas, m_1, m_2, $\cdots$, m_n, Dempster's rule is defined as

$$[m_1 \oplus m_2 \cdots \oplus m_n](\theta) = \begin{cases} 0, \quad \theta = \phi, \\ \dfrac{\sum\limits_{A_1 \cap A_2 \cdots \cap A_n = \theta, A_1, A_2, \cdots, A_n \subseteq \Theta} m_1(A_1) m_2(A_2) \cdots m_n(A_n)}{\sum\limits_{A_1 \cap A_2 \cdots \cap A_n \neq \phi, A_1, A_2, \cdots, A_n \subseteq \Theta} m_1(A_1) m_2(A_2) \cdots m_n(A_n)}, \quad \theta \neq \phi, \end{cases}$$

under the condition that $\sum\limits_{A_1 \cap A_2 \cdots \cap A_n \neq \phi} m_1(A_1) m_2(A_2) \cdots m_n(A_n) \neq 0$.

2.2. Evidential reasoning rule

Yang and Xu (2013) extended DS theory and proposed a refined ER rule that proceeds along the following steps. First, the original evidence is transformed into modified evidence using relative weights as follows:

$$\begin{cases} M_i(\theta) = w_i m_i(\theta), \quad i = 1, 2, \cdots, n; \theta \subset \Theta, \\ M_i(\Theta) = \overline{M}_i(\Theta) + \widetilde{M}_i(\Theta) = 1 - w_i \sum\limits_{\theta \subset \Theta} m_i(\theta), i = 1, 2, \cdots, n, \\ \widetilde{M}_i(\Theta) = w_i (1 - \sum\limits_{\theta \subset \Theta} m_i(\theta)), \quad i = 1, 2, \cdots, n, \\ \overline{M}_i(\Theta) = 1 - w_i, \quad i = 1, 2, \cdots, n. \end{cases} \tag{1}$$

Note that the probability mass assigned to the whole set, $M_i(\Theta)$, which denotes the degree of ignorance, is split into two parts: