Expert Systems With Applications 77 (2017) 136–147

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Heuristics for the robust vehicle routing problem with time windows

Simen Braaten[a], Ola Gjønnes[a], Lars Magnus Hvattum[b], Gregorio Tirado[c,*]

[a] Norwegian University of Science and Technology, Norway
[b] Faculty of Logistics, Molde University College, Norway
[c] Universidad Complutense de Madrid, Spain

ARTICLE INFO

Article history:
Received 21 July 2016
Revised 27 December 2016
Accepted 25 January 2017
Available online 1 February 2017

Keywords:
Robust optimization
Metaheuristic
Uncertainty
Travel time

ABSTRACT

Uncertainty is frequently present in logistics and transportation, where vehicle routing problems play a crucial role. However, due to the complexity inherent in dealing with uncertainty, most research has been devoted to deterministic problems. This paper considers a robust version of the vehicle routing problem with hard time windows, in which travel times are uncertain. A budget polytope uncertainty set describes the travel times, to limit the maximum number of sailing legs that can be delayed. This makes sure that improbable scenarios are not considered, while making sure that solutions are immune to delays on a given number of sailing legs. Existing exact methods are only able to solve small instances of the problem and can be computationally demanding. With the aim of solving large instances with reduced running times, this paper proposes an efficient heuristic based on adaptive large neighborhood search. The computational study performed on instances with different uncertainty levels compares and analyzes the performance of four versions of the heuristic and shows how good quality solutions can be obtained within short computational times.

1. Introduction

Vehicle routing problems play a crucial role in logistics and transportation, and many different variants have been widely studied in the literature during the last decades (Toth & Vigo, 2014). Most of this research has been devoted to deterministic problems, where all the parameters related to the problem data are known with certainty beforehand. However, in real life it is common to observe uncertainty, which decision makers must take into consideration.

The motivation of the optimization problem faced in this paper stems from the maritime shipping industry, where routing decisions are complicated by frequent delays that may severely affect route plans and delivery times. Weather conditions, port congestion or strikes, and mechanical problems are possible reasons for ship delays. As a consequence of the high operating cost of the ships, a pure cost minimization perspective means that no buffers are imbedded in the schedules, and any delays imply a need for reoptimizing the schedules. Given the massive importance of maritime transportation to the world's economy (Christiansen, Fagerholt, Nygreen, & Ronen, 2013), improving the efficiency of this sector has significant positive effects, and facilitates further growth.

This paper considers a *robust vehicle routing problem with time windows* (RVRPTW) that arises in this context. Even though maritime transportation was the main motivation of our work, the robust vehicle routing problem has many applications in many other areas of logistics in which delays are frequent. The robustness implies that the solution should be immune to uncertainty in data. The relevance of introducing robust optimization solutions was stressed by Ben-Tal and Nemirovski (2000), who concluded that, in real-world applications of linear programming, the possibility that a small uncertainty in the data can make the optimal solution meaningless from a practical viewpoint, cannot be ignored. In our case, the travel times of the vehicles will be taken as uncertain parameters, considering several possible sets of combinations of delays. This leads to a more realistic setting, in which the proposed solutions will remain feasible even for realizations of the travel times that differ from the expectation.

Agra et al. (2012, 2013) developed exact solution methods for the RVRPTW, but these methods are quite time consuming and can only be used to solve small instances. The main contribution of this paper is an efficient heuristic, based on adaptive large neighborhood search (ALNS), that is able to provide very good quality solutions for large RVRPTW instances. To design the heuristic, the concept of *programming by optimization* (Hoos, 2012) is used. That is, the heuristic is implemented by adding a large range of search

* Corresponding author.
E-mail addresses: simenval@stud.ntnu.no (S. Braaten), olagjo@stud.ntnu.no (O. Gjønnes), Lars.M.Hvattum@himolde.no (L.M. Hvattum), gregoriotd@mat.ucm.es (G. Tirado).

http://dx.doi.org/10.1016/j.eswa.2017.01.038
0957-4174/

components, possibly with many parameters that influence their individual behavior. Instead of manually testing the effectiveness of each component, or selecting components by trial and error, an automated algorithmic optimization procedure is used to determine the best functioning combination of search components. To this end, the irace software for parameter tuning is used (Lopez-Ibanez, Dubois-Lacoste, Stützle, & Birattari, 2011). The presentation of a successful application of the concept of programming by optimization, which is not widely used in the literature when approaching similar problems, is also an interesting contribution of the paper. Additionally, we test four different versions of the heuristic, to evaluate the effect of three important design choices: one version of the heuristic does not include a proposed preprocessing phase, another version excludes all local search components, and a third version uses a more common move acceptance rule. The main version, on the other hand, includes both preprocessing, local search, and an alternative move acceptance rule. The computational study performed shows how these three new elements provide significant improvements and thus they are important contributions of the paper as well.

The remainder of this paper is structured as follows. The next section defines the problem at hand, and introduces its mathematical formulation. Section 3 presents a brief review of relevant literature. The methodology proposed to solve the problem, including preprocessing, destruction, insertion and local search heuristics, an efficient feasibility check method, and an overarching ALNS heuristic, is introduced in Section 4. A detailed computational study is presented in Section 5, and finally, some concluding remarks and suggestions for future work are discussed in Section 6.

2. Problem description

The RVRPTW is a generalization of the well-known *vehicle routing problem with time windows* (VRPTW). The RVRPTW results from a special case in maritime shipping, where goods are transported between given pairs of loading and unloading locations. Whenever the goods are transported using full loads, the problem can be modelled using a graph where nodes represent the loading operation, the movement to the unloading location, and the unloading operation. Additional nodes represent the starting points of empty vehicles and the artificial final positions of the vehicles after completing their itineraries. Arcs between nodes represent travel from the previous unloading operation (or the starting position) to the next loading operation (or the artificial final position). There is a time window associated to the loading operation of each transportation task, and the set of vehicles is typically heterogeneous, with vehicles varying in size and speed.

Full load pick-up and delivery problems therefore reduce to instances of an asymmetric, heterogeneous fleet VRPTW. Let us further consider the possibility of delays happening between the loading operation and the arrival at the next loading operation. It is important to hedge against these delays, and the company may want a robust schedule such that even when a few delays happen, the itineraries of the vehicles are still not in violation of any time windows. This leads to the RVRPTW.

A mathematical formulation of the RVRPTW was first proposed in Agra et al. (2012). We present a slightly adapted mathematical model to clarify the problem description. To model the uncertainty in travel times in the presence of hard time windows, a step-wise (layered) formulation is used. Let us denote the set of nodes by N, using i and j to denote general nodes, and let N^* be the subset of N that excludes the origin o and destination d. The set of arcs is denoted as A and contains pairs of nodes, (i, j). The set of vehicles is called V with elements v. Now we can assign to each vehicle v a cost c_{ijv} for traversing edge (i, j), and to each node i a time window $[a_i, b_i]$. Then x_{ijv} are binary decision variables that take the value 1 if vehicle v uses the edge (i, j).

To account for the robustness and the time windows, some additional notation is required. First, to simplify the notion of visiting the nodes in an order, we introduce steps s from 0 and up to some upper bound $\bar{S}$ on the length of a path, for example $\bar{S} = |N|$. These steps are a different interpretation of the layers used in Agra et al. (2012). With these in place, we define $\mathcal{A}^S$ as the set of valid edge-step combinations. For example, in the 0'th step, only edges going out from o can be used, as we always need to start in the origin. Thus, for the 0'th step, $\mathcal{A}^S$ only contains 3-tuples such as $(o, i, 0)$, meaning that a vessel can go from the origin to node i in step 0. This allows us to consider the binary decision variable y_{ijvs}, which is 1 if and only if vessel v goes from i to j as the s'th step. Second, to handle the robustness, we introduce the set $\mathcal{T}_v^\Gamma$ of delay patterns p that a vessel v can face, each with a maximum of Γ delays. With this, we can define t_{ijvp} as the travel time for v along (i, j) in delay pattern p. The proposed model is the following:

Sets

N	- Nodes
N^*	- $N \setminus \{o, d\}$
A	- Arcs
$\mathcal{A}^S$	- Allowable arc/step combinations
V	- Vehicles
$\mathcal{T}_v^\Gamma$	- Delay patterns for v

Indices

i, j	- Node
v	- Vehicles
s	- Step
p	- Delay pattern

Parameters

$\bar{S}$	- Maximal number of steps
c_{ijv}	- Cost for vehicle v of travelling edge (i, j)
t_{ijvp}	- Travel time for vehicle v of travelling edge (i, j) facing delay pattern p
o	- Origin node
d	- Destination node
a_i	- Time window opening at node i
b_i	- Time window closing at node i

Decision variables

x_{ijv}	- Flag variable for vehicle v travelling edge (i, j)
y_{ijvs}	- Flag variable for vehicle v travelling edge (i, j) as its step number s

Objective function and restrictions

$$\min \quad z = \sum_{v \in V} \sum_{(i,j) \in A} c_{ijv} x_{ijv} \tag{1}$$

$$\text{s.t.} \quad \sum_{v \in V} \sum_{j|(i,j) \in A} x_{ijv} = 1, \quad i \in N^*, \tag{2}$$

$$\sum_{j|(j,i,s-1) \in \mathcal{A}^S} y_{jiv,s-1} - \sum_{j|(i,j,s) \in \mathcal{A}^S} y_{ijvs} = \begin{cases} -1 & \text{if } (i = o) \\ 1 & \text{if } (i = d \quad \text{and} \quad s = \bar{S}), \\ 0 & \text{otherwise} \end{cases} \qquad \begin{array}{l} 1 \le s \le \bar{S}, \\ i \in N_s, v \in V, \end{array} \tag{3}$$

$$\sum_{(i,j,s) \in \mathcal{A}^S} y_{ijvs} = x_{ijv}, \quad (i, j) \in A, v \in V, \tag{4}$$