Expert Systems With Applications 69 (2017) 189–200

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

Finite length white noise generation with an immuno inspired algorithm

Mateus Giesbrecht*, Celso Pascoli Bottura

School of Electrical and Computing Engineering FEEC, University of Campinas UNICAMP, Av. Albert Einstein, 400, Cidade Universitária Zeferino Vaz, Campinas, SP, Brasil

ARTICLE INFO

Article history:
Received 2 August 2016
Revised 11 October 2016
Accepted 11 October 2016
Available online 21 October 2016

Keywords:
White noise
White noise generation
Immuno-inspired algorithms

ABSTRACT

In this paper an immuno-inspired algorithm is proposed to generate sequences of data close to the ideal white noise. The motivation to propose the algorithm is that in many cases there is a necessity to generate white noises with a small number of samples, and the pseudo-random generators may fail to perform this task. The proposed algorithm is based on the maximization of the whiteness criterion, clearly defined in this paper, and is different from other immuno-inspired algorithms because it presents an automatic regulation of the suppression threshold, that is an important control variable of the algorithm. This feature allows the proposed algorithm to reach good results, even for different sizes of candidate solutions.

To test the proposed algorithm, the results obtained from it are compared to the results obtained from a known pseudo-random generator and it is shown that the solutions obtained with the proposed algorithm are better, for time and frequency domain, if the number of samples required is limited. It is also shown that the proposed algorithm consumes a time comparable to the pseudo-random generator to reach solutions that are better than the ones obtained with that kind of algorithm.

1. Introduction

A time series, or a stochastic process, is a family of random variables indexed by a scalar that represents the time. In the discussion developed in this paper, this scalar is discrete and is denoted by k. To belong to the same stochastic process, the random variables for all different k must be defined in the same probability triple $\{\Omega, \mathcal{F}, P\}$, where Ω is the sample space, $\mathcal{F}$ is a σ-algebra defined on the subsets of Ω and P is a probability measure. More details about those hypothesis and definitions can be found in standard statistics books such as (Mortensen, 1987) or (Papoulis & Pillai, 2002). A sequence of numbers sampled along the time that satisfies the statistical rules that describe a specific stochastic process is defined as a realization of that stochastic process.

A special kind of stochastic process is defined as white noise. By definition, the white noise is a stochastic process that follows a Gaussian distribution with zero mean and a positive definite variance. In this process, each sample is uncorrelated to any other sample. This time domain characteristic results in a flat spectrum in the frequency domain. Thus, the name of this time series comes from the analogy between its frequency domain characteristic and the white light, that is a light which spectrum has the same intensity for all frequencies.

In stochastic calculus context, the white noise is defined as the derivative of the Wiener process, that describes the position of a particle subject to Brownian motion. It can be proved that the ideal white noise is not realizable (Kloeden & Platen, 1992), that is, it is impossible to create an exact realization of that stochastic process.

Although not realizable, the white noise plays an extremely important role in stochastic process theory (Kloeden & Platen, 1992), (Oksendal, 2003), system identification (Ljung, 1999), (Katayama, 2005), signal processing (Oppenheim & Schafer, 2010), time series analysis (Durbin & Koopman, 2001) and many other theories, since this signal can be used as the input to filters, resulting in outputs with any desired set of auto-covariance matrices, that are directly related to the spectrum (Box, Jenkins, & Reinsel, 2008). The output auto-covariance matrices depend only on filter parameters. From this point of view, the white noise can be considered as the stochastic equivalent to the deterministic impulse function, from which any deterministic signal can be generated if this function is applied to an appropriate dynamical system.

The problem of finding a model that generates a sequence of data with any desired set of auto-covariance matrices (or any spectrum) from a white noise input is defined as time series realization problem (Anderson, Moore, & Loo, 1969), (Gevers & Kailath, 1973).

* Corresponding author.
E-mail addresses: mateus@fee.unicamp.br (M. Giesbrecht), bottura@fee.unicamp.br (C. Pascoli Bottura).

http://dx.doi.org/10.1016/j.eswa.2016.10.023

In this problem, the desired sequence of data may be unidimensional or multidimensional. In the first case, the methods based on auto-regressive models or frequency response are adequate (Box et al., 2008), (Young, 2011). In the second case, it is convenient to work with state space models, that can be easily generalized for any dimension. There are many approaches to solve the time series realization problem in state space, such as the one proposed by Faurre, that is based on solving a linear matrix in equation that comes from the desired state space model and the desired set of auto-covariances (Faurre, 1967), the one proposed by Aoki, that uses the desired auto-covariance data to determine a Riccati equation, from which the solution can be obtained (Aoki, 1987), the one proposed by Akaike, that is based on canonical correlations (Akaike, 1974), the one proposed by Durbin and Koopman, based on maximum likelihood estimation (Durbin & Koopman, 2001) and many others. Although those methods are based on different approaches, all have in common that the results are the model matrices and the variance for a white noise to be given as input to the model to generate realizations of the desired time series. Normally, in those cases, the input white noise is multivariate, what means that insted of a Gaussian sequence of uncorrelated numbers, the desired signal is a Gaussian sequence of uncorrelated vectors.

From the discussion above, the importance of a method to generate good approximations for unidimensional and multidimensional white noise realizations is clear. Usually, if the number of desired samples is in an order of magnitude of 10^3 or greater, pseudo-random generators as the one proposed in Marsaglia and Tsang (1984) are good sources of approximations for white noise realizations. If this hypothesis is not verified, the sequences obtained from pseudo random generators are not good approximations for the white noise, as demonstrated in Giesbrecht and Bottura (2011). In that reference, an immuno-inspired method is proposed to generate white noise realizations. This method is based on transforming the white noise generation into an optimization problem, and solving it with an immuno-inspired algorithm. It was shown that the new method produced white noise realizations with better quality than the ones produced by a pseudo-random generator. The sequences of data created with the two different approaches were used as inputs to a model determined with the Aoki method (Aoki, 1987) and the outputs were closer to the desired time series when the white noise obtained with the immuno-inspired method was applied.

In this article, a new implementation of the algorithm introduced in Giesbrecht and Bottura (2011) is presented. This new approach differs from the original one by the introduction of an automatic procedure to regulate a crucial control parameter of the algorithm, that is the suppression threshold. Also, in this paper a whiteness criterion is clearly stated. This criterion is useful to determine how close any sequence of data is from the ideal white noise and can be used as a measure of the quality of the signals created by any white noise generation method.

The algorithm proposed in this paper was tested and the results were compared to the pseudo-random generator proposed in Marsaglia and Tsang (1984) for many dimensions and quantity of samples. This comparison allows to conclude about the limits of the number of samples and the dimension where a method is superior to the other. The spectra of the realizations obtained with the proposed method and the ones obtained with normal pseudo-random generators are also compared.

This paper is structured as follows: In the next section the mathematical definitions of the unidimensional and the multidimensional white noises in time and frequency domains are presented. The difficulties to generate a finite length white noise realization are also discussed. From the definitions, the whiteness criterion is introduced. In the third section the immuno-inspired algorithms are discussed. This class of algorithms presents interesting characteristics to solve optimization problems and is used as basis to the algorithm presented in this paper. In the fourth section the proposed algorithm is introduced. In the fifth section the results of the tests executed to evaluate the quality of the solutions obtained with the algorithm are presented. Finally in the sixth section the conclusions of this work are shown, closing the article.

2. White noise

In this section the definitions of unidimensional and multidimensional discrete white noises in time and frequency domains are presented. In the sequence, the difficulties related to the generation of finite length white noises are discussed. To finish this section, a whiteness criterion is introduced. This criterion is used to evaluate which one of two different signals is closer to the ideal white noise definition and is the key to understand the method presented in this paper.

2.1. Unidimensional discrete white noise in time domain

Let $x(k) \in \mathbb{R}$, $(k \in \mathbb{Z} | -\infty < k < \infty)$, be an infinite realization of an unidimensional discrete time series. Its auto-covariances $\Lambda_x(t)$ are defined as:

$$\Lambda_x(t) = E[(x(k) - \mu_x)(x(k+t) - \mu_x)], \quad t \in \mathbb{Z} \tag{1}$$

where $E[\bullet]$ is the expectation operator, t is an integer defined as lag and $\mu_x \in \mathbb{R}$ is the mean of the time series realization $x(k)$ defined as:

$$\mu_x = E[x(k)] = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{k=-N}^{N} x(k) \tag{2}$$

By definition, a realization of an ideal infinite unidimensional Gaussian white noise $e(k)$ is a sequence of numbers that follows a normal distribution, has zero mean and presents a set of auto-covariances that satisfies the following relation:

$$\Lambda_e(t) = \begin{cases} \sigma^2 & t = 0 \\ 0 & t \neq 0 \end{cases}, \quad t \in \mathbb{Z} \tag{3}$$

where $\Lambda_e(t)$ denotes the auto-covariance of the signal $e(k)$ for the lag t and $\sigma^2 \in \mathbb{R}$ is its variance.

2.2. Multidimensional discrete white noise in time domain

In the multidimensional time series analysis, it is convenient to define a white noise with dimension greater than one. This definition is similar to the unidimensional one but, instead of using unidimensional samples defined in the space $\mathbb{R}$, the samples are defined in the $\mathbb{R}^l$ space. Consequently, the mean becomes a $\mathbb{R}^l$ vector and the auto-covariances become $\mathbb{R}^{l x l}$ matrices.

Let $x(k) \in \mathbb{R}^l$, $(k \in \mathbb{Z} | -\infty < k < \infty)$, be an infinite realization of an l-dimensional discrete time series. Its auto-covariance matrices $\Lambda_x(t)$ are defined as:

$$\Lambda_x(t) = E[(x(k) - \mu_x)(x(k+t) - \mu_x)^T], \quad t \in \mathbb{Z} \tag{4}$$

where $E[\bullet]$ is the expectation operator, $\bullet^T$ is the matrix transpose operator, t is an integer defined as lag and $\mu_x \in \mathbb{R}^l$ is the mean of the time series realization $x(k)$ defined similarly as in the unidimensional case.

A realization of an ideal l-dimensional Gaussian white noise $e(k)$ is a sequence of l-dimensional vectors that follows a l-dimensional normal distribution, has zero mean and presents a set of auto-covariances that satisfies the following relation:

$$\Lambda_e(t) = \begin{cases} \Sigma^2 & t = 0 \\ \mathbf{0}_{lXl} & t \neq 0 \end{cases} \quad t \in \mathbb{Z} \tag{5}$$