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Uncertainty quantification of squeal instability under two fuzzy-interval cases

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Received 15 October 2016; received in revised form 19 March 2017; accepted 5 July 2017

Abstract

Automotive brake systems are always subjected to various kinds of uncertain factors and the uncertainty analyses of brake squeal have been investigated for decades. However, most of the existing methods of squeal analysis are merely effective in tackling single uncertain case. In this study, a unified method is developed for the uncertainty quantification of disc brake squeal, which is able to handle two fuzzy-interval cases. Due to subjective opinions or limited data, in the first fuzzy-interval case, the uncertain brake parameters are expressed as either fuzzy variables or interval variables, which exist in brake systems simultaneously and independently; whereas, the uncertain brake parameters are all treated as interval variables, but their lower and upper bounds are expressed as fuzzy variables in the second fuzzy-interval case. In the developed method, the first fuzzy-interval case is equivalently treated as a special case of the second one. On the basis of fuzzy-boundary interval variables, an uncertainty quantification model under two fuzzy-interval cases is established, in which the unified uncertain response is computed with the aid of the combination of level-cut strategy, Taylor series expansion, subinterval analysis and Monte Carlo simulation. The unified method is then extended to quantify the uncertainty in disc brake squeal analysis. The studies of several numerical examples demonstrate that the proposed method is able to quantify the uncertainty of squeal instability effectively. In addition, the proposed method can also help to carry out reliability analysis and optimization to reduce the likelihood of squeal occurrence.

Keywords: Brake squeal; Uncertainty quantification; Fuzzy-interval uncertainty; Fuzzy variable; Interval variable

1. Introduction

1.1. Fuzzy-interval analysis

Uncertainty quantification plays a more and more important role in the structural response analysis. In order to cope with uncertainty, both probabilistic and non-probabilistic methods can be applied. Among the available non-

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http://dx.doi.org/10.1016/j.fss.2017.07.006

Please cite this article in press as: H. Lü et al., Uncertainty quantification of squeal instability under two fuzzy-interval cases, Fuzzy Sets Syst. (2017), http://dx.doi.org/10.1016/j.fss.2017.07.006

probabilistic methods, fuzzy set theory [1] and interval theory [2] have been widely used to quantify the influence of uncertainty on structural response in the last decades.

The fuzzy set theory, which is introduced by Zadeh [1], provides a non-probabilistic way to represent uncertainty using membership functions. The membership functions of fuzzy parameters are generally constructed based on the available expert opinions. The interval theory is another efficient method to expresses non-probabilistic uncertainty. The interval analysis, which is mainly based on the work of Moore [2], describes uncertain structural parameters as interval variables with well-defined lower and upper bounds. The traditional structural response analysis under fuzzy uncertain case or under interval uncertain case has undergone a rapid development. In most of traditional researches, the fuzzy method and the interval method are separately applied to conduct the non-probabilistic analysis of structural response. However, the complicated mechanical structures in engineering, such as automotive disc brakes, may be subjected to two or more uncertain cases simultaneously. In some special situations, fuzzy uncertainty and interval uncertainty are likely to exist in engineering structures simultaneously. To cope with those special situations, two fuzzy-interval cases have been taken into account for structural response analysis. The first one is the fuzzy and interval uncertainty case [3], in which fuzzy parameters and interval parameters exist in structures simultaneously and independently. The second one is the fuzzy-boundary interval uncertainty case [4], in which uncertain parameters of structures are all treated as interval variables while their lower and upper bounds are expressed as fuzzy numbers rather than deterministic numbers due to subjective knowledge.

Some significant developments have been achieved in the uncertainty analysis fields under either the fuzzy and interval uncertainty case, or the fuzzy-boundary interval uncertainty case [3,4]. Recently, the unified analysis of uncertain problems under two uncertain cases has attracted increasing interest in the response analysis of acoustic field [5]. In [5], an interesting unified analysis of the acoustic problem with probability and interval uncertainties is proposed based on an inverse mapping hybrid perturbation method. Nevertheless, the unified numerical methods, which can be used for structural response analysis under the two fuzzy-interval cases, have not been explored and reported at present. Thus, it is significant to develop more unified frameworks for the uncertainty quantification of structural response under multiple uncertain cases, especially under the fuzzy-interval cases.

1.2. Uncertainty quantification of brake squeal

Friction-induced vibrations have already attracted great attention of researchers since the 1960s. The friction-induced vibrations are responsible for many acoustical problems, such as the noise of rail-wheel contact in railway industry [6], the squeaking noise in hip endoprosthesis [7] and the brake squeal in automotive engineering [8]. Automotive brake squeal has received the most attention due to high warranty costs and consistent complaints. Brake squeal generally occurs in the frequency range from 1 to 16 kHz [9] and its sound pressure level is usually equal to or greater than 78 dB [10].

Several mechanisms have been put forward to explain the generation of brake squeal. Those mechanisms can be summarized as stick-slip [11], sprag-slip [12], hammering excitation [13], mode coupling [14], moving loads [15], time delay [16], and so on. However, so far, there has been no general consensus on the real cause of squeal phenomenon due to its highly fugitive nature. A rich number of early studies on brake squeal have been summarized in several review papers [17–20].

Two popular approaches are available for brake squeal analysis: the linear complex eigenvalue analysis (CEA) and the nonlinear transient dynamic analysis (TDA) [21]. The advantages and disadvantages of CEA and TDA on predicting brake squeal have been deeply discussed by Ouyang et al. [18]. CEA is based on the linearization of nonlinear equations of motion and it is carried out around an equilibrium point. In CEA, system stability is studied in Lyapunov's sense and positive real parts of complex eigenvalues indicate the instability [22]. CEA can provide a set of unstable frequencies but cannot give the instability behaviors. In one run of CEA, all unstable frequencies of a brake can be found under the given operating conditions. The high efficiency on predicting unstable frequencies is the greatest advantage of CEA. However, CEA may lead to an under-estimation or over-estimation of unstable frequencies due to the neglect of nonlinear effects as previously reported by Sinou et al. [23,24]. On the contrary, TDA permits to obtain the evolution of mechanical quantities versus time conveniently. The nonlinear effects of contact with friction can be fully considered in TDA [25]. However, the computational time of TDA is prohibitive. Indeed, the nonlinear analyses of squeal events based on TDA are quite complicated and challenging tasks. In recent literature, the influence of ramp loading conditions on the transient analysis of brake squeal [26], the transient simulation of friction-induced vibra-

Please cite this article in press as: H. Lü et al., Uncertainty quantification of squeal instability under two fuzzy-interval cases, Fuzzy Sets Syst. (2017), http://dx.doi.org/10.1016/j.fss.2017.07.006