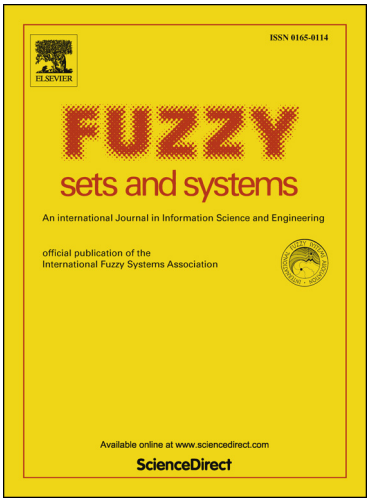
Accepted Manuscript



Hyper effect algebras

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PII: S0165-0114(16)30453-5
DOI: http://dx.doi.org/10.1016/j.fss.2016.12.012
Reference: FSS 7145

To appear in: *Fuzzy Sets and Systems*

Received date: 6 May 2016
Revised date: 13 December 2016
Accepted date: 14 December 2016

Please cite this article in press as: A. Dvurečenskij, M. Hyčko, Hyper effect algebras, *Fuzzy Sets Syst.* (2016), http://dx.doi.org/10.1016/j.fss.2016.12.012

HYPER EFFECT ALGEBRAS

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Abstract. We present hyper effect algebras as a generalization of effect algebras. The result of the hyper summation of two mutually excluding events is not an element of the algebra but rather a subset (not necessarily a singleton) of the algebra. We present basic notions like states on hyper effect algebras. We present two standard examples of hyper effect algebras starting from effect algebras. We show how we can effectively generate finite models of hyper effect algebras and we point out problems with associativity. Finally, we provide a representation of any finite linearly ordered hyper effect algebra.

1. Introduction

Effect algebras are important algebras of the theory of quantum structures. They were introduced by Foulis and Bennett [FoBe] as partial algebras with a partially defined operation $+$, where $a + b$ means disjunction of two mutually excluding events a and b. Effect algebras generalize Boolean algebras, orthomodular posets, orthomodular lattices, MV-algebras, etc. The most important example of effect algebras, which is crucial for mathematical foundations of quantum mechanics, is the system $\mathcal{E}(H)$ of all Hermitian operators A on a Hilbert space H that are between the zero operator, O, and the identity, I. Then the events of $\mathcal{E}(H)$ have a fuzzy character, i.e. their spectra are subsets of the real interval $[0, 1]$. Every MV-algebra can be studied also in the framework of effect algebras if we define a partial operation $+$ on an MV-algebra as follows $a + b$ is defined iff $a \odot b = 0$, and in such a case, $a + b := a \oplus b$. In addition, every lattice ordered effect algebra can be covered by a system of blocks which are MV-algebras, [Rie]. A crucial notion for effect algebras is a state which is a $[0, 1]$-valued finitely additive functional on the effect algebra preserving $+$ and 1. It is an analogue of a finitely additive probability measure. More about effect algebras can be found in the monograph [DvPu] and more about MV-algebras can be consulted with [Cha, CDM].

The theory of hyper-structures, which is a generalization of the concept of algebraic structures, was firstly introduced by Marty in [Mar] in 1934. It has been

[1]Keywords: Effect algebra, MV-algebra, hyper effect algebra, state, h-state, sup state, hyper state, associativity, models of hyper effect algebras

AMS classification: 03G12, 06D35

This work was supported by the grant VEGA No. 2/0069/16 SAV and GAČR 15-15286S.