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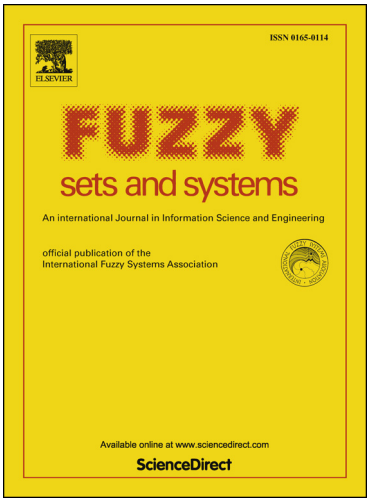

Left and right distributivity equations for semi-t-operators and uninorms

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Left and right distributivity equations for semi-t-operators and uninorms

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Abstract

Recently, the distributivity equation for different kind of operators has become a focus of research as the crucial tool in the numerous applications as utility and optimization theory, or integration theory. The main purpose of this paper is to study the distributivity equations between two special classes of aggregation operators namely, semi-t-operators and uninorms. Without the commutativity assumption of semi-t-operators it is necessary to consider the left and the right distributivity conditions separately. In both cases, the obtained solutions are significantly different.

Keywords: Aggregation operator, Distributivity equations, Idempotent operation, Semi-t-operator, Uninorm.

1. Introduction

Investigation of the problem of distributivity was posed in [1], pp. 318-319 and, in a recent time, it has been directed toward many different areas, such as aggregation functions [4], fuzzy implications [2], uninorms and nullnorms [14, 20, 21, 24, 25], semi-nullnorms and semi-t-operators [9, 10, 29, 31].

In this study, our aim is to obtain algebraic structures with weaker assumptions than nullnorms and t-operators fulfilling left or right distributivity equation. The characterization is interesting from a theoretical point of view and also in terms of their applications because they have proved to be useful in several fields, such as decision making theory [12], integration theory [28], fuzzy logic framework [18] and others. This research also complements certain results of our previous studies [9, 10, 11].

The present communication is organized as follows. In Section 2, we introduce some of the weak algebraic structures. Then we recall basic facts about the distributivity equations (Section 3). In Section 4, we state the main results, which are characterization theorems of the left and the right distributivity equations from described classes of aggregation operators, thereby generalizing the results of [6, 11, 13, 20, 29]. In the last section, we provide some useful examples and a summary of our research.

2. Associative, monotonic binary operations

In this section, we recall some basic definitions and facts about some special classes of aggregation operators (see [5]).

Definition 1 ([17])**.** A triangular semi-norm T is an increasing operation $T : [0,1]^2 \to [0,1]$ with neutral element 1.
A triangular semi-conorm S is an increasing operation $S : [0,1]^2 \to [0,1]$ with neutral element 0.
A triangular norm T is a commutative, associative triangular semi-norm.
A triangular conorm S is a commutative, associative triangular semi-conorm.

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