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The orness value for rank-dependent welfare functions and rank-dependent poverty measures

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Abstract

We propose two *distribution-sensitivity* criteria to classify the rank-dependent welfare functions. These criteria compare the reaction of the welfare function to *lossy* transfers and *lossy equalization* transfers among individuals. We see that these classifications in terms of their *distribution-sensitivity* to these transfers can be established focusing only on the weights assigned to each welfare function. We also propose a criterion to sort the rank-dependent welfare functions and the rank-dependent poverty measures in terms of a mathematical value called *orness*. We provide a classification in terms of the *orness* value for the welfare functions of the S-Gini family, the Bonferroni index and the De Vergottini index. Another classification is provided for the poverty measures of the Poverty Gap Ratio, the Sen indices, the Thon index and the Thon family of indices, the Kakwani family of indices and the S-Gini family of indices. Finally, we prove that the *orness* classification for welfare functions and the *orness* classification for poverty measures can be interpreted as a *distribution-sensitive* classification since they have a direct link with the classifications proposed above. Moreover, we see that for a subset of welfare functions and another subset of poverty measures, the *orness* classification and the *distribution-sensitivity* classification based on *lossy* transfers and *lossy equalization* transfers are equivalent.

Keywords: Aggregation functions; OWA operators; Orness; Rank-dependent poverty measure

1. Introduction

In the economic field, it is thoroughly accepted that every income poverty measure should decrease with an income transfer from a poor person to another poorer person. Besides, a welfare function should increase with an income transfer from a better-off individual to a worse-off individual (see Watts [27] and Sen [23]). Therefore, it is widely accepted that welfare functions and poverty measures should take into account distribution-sensitivity (see Chakravarty [10] and Zheng [33]). However, the degree of *distribution-sensitivity* of these transfers is minimal since they do not give up any income in return for the distribution improvement produced by the transfer. It is also well-known that

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some poverty measures can sacrifice larger income values than others in return for a given distributional improvement. In the literature, the degree of *distribution-sensitivity* of welfare functions and poverty measures has always be regarded as an important factor. Nevertheless, there are few formal definitions of *distribution-sensitivity* comparisons of these measures. Zheng [34] provides a theoretical base to compare *distribution-sensitivity* for a class of additively separable poverty indices. However, as stressed by Bosmans [7], the criterion introduced by Zheng does not allow comparisons among rank-dependent poverty measures. For this reason, Bosmans [7] introduces a condition based on the dominance of the vector of weights that allows comparisons between rank-dependent poverty measures in terms of their *distribution-sensitivity* to some specific transfers.

In the poverty field, a *lossy transfer* among the poor is a transfer in which the better-off poor individual gives an amount, whereas the worse-off poor individual receives a lower amount (see Okun [21]). On the other hand, a *equalization transfer* among the poor entails a redistribution of the poor incomes, assigning the same income value to all poor people, with the implication of a loss in the total income of the poor. Now, according to the criterion of *lossy transfer* (*equalization transfer*) among the poor, a poverty measure P_W would be at least as *distribution-sensitive* as a welfare measure P_V if P_W accepts each *lossy transfer* (*equalization transfer*) among the poor that P_V does. In order to check these *distribution-sensitive* criteria Bosmans [7] proposes an easy-to-check condition based on the weights of the rank-dependent poverty measures that allows us to rank several members of the class.

The first objective of this paper is to propose a similar *distribution-sensitivity* criteria for welfare functions. A *lossy transfer* on welfare functions is a transfer in which the better-off individual gives an amount β, whereas the worse off individual receives only an amount of $\alpha < \beta$. Similarly, an *equalization transfer* on welfare functions entails a redistribution of all the incomes, assigning the same income value to all of individuals, with the implication of a loss in the total income. Then, according to the *lossy transfer* (*equalization transfer*) criterion of *distribution-sensitivity*, a welfare measure W would be at least as *distribution-sensitive* as a welfare measure V, if W accepts each *lossy transfer* (*equalization transfer*) that V does. Similarly to poverty measures, we will be able to rank welfare functions in terms of these distribution-sensitivity criteria, focusing on the form of the weights assigned to each welfare function.

The second aim of the paper is to propose a new *distribution-sensitivity* criterion based on a mathematical value called *orness*, which can be assigned to every rank-dependent welfare function and every rank-dependent poverty measure. The *orness* value will allow us to rank all the members of the two classes.

The *orness* value is a numerical value that is assigned to every ordered weighted averaging (hereafter OWA) operator. The OWA operators were first introduced in decision theory by Yager [30] as a new aggregation technique that collects the multiple criteria to form an overall decision function. In recent years, OWA operators have received great attention and scholars have applied them in different contexts, such as decision making under uncertainty, fuzzy system, welfare and so on (see Yager and Kreinovich [32], Fodor and Roubens [14], Yager [31], García-Lapresta et al. [16], Aristondo et al. [2] and [3] and Aristondo and Ciommi [4]). As mentioned, every OWA operator has assigned to it a numerical value called *orness*. The *orness* of an OWA operator was also introduced by Yager [30] in order to classify OWA operators with regard to their location between two extreme situations. The first one, the *or*, means full compensation among criteria and the last one, the *and*, means that a higher degree of satisfaction of one of the criteria can compensate for a lower degree of satisfaction of another.

We need to assign an *orness* value to each member of the class of rank-dependent welfare functions and the class of rank-dependent poverty measures. Hence, first of all we need to rewrite all of them in terms of an OWA operator. The class of the family of rank-dependent social welfare functions was originally defined by Yaari [29] as the weighted average of the variable of interest, where the weight decreases with the position of the variable in the distribution (see Aaberge et al. [1]). Rank-dependent welfare functions are not directly OWA operators, but they can easily be rewritten as OWA operators simply restricting the domain to 0 and 1. Then, we are able to assign an *orness* value to each welfare function and to classify them in terms of their *orness* value. In fact, we provide a classification based on the *orness* value for the welfare functions of the S-Gini class introduced by Weymark [28], the Bonferroni [6] welfare function and the De Vergottini [12] index.

With respect to the rank-dependent poverty measures, these indices are defined on the basis of the difference between a poverty line and its income level, namely, the poverty gap, for which the weight attached to each individual depends on its position in the distribution. We see that all the rank-dependent poverty measures can be rewritten as the product of a normalization factor and an OWA operator. Hence, we are able to assign an *orness* value to every rank-dependent poverty measure and offer a classification in term of this *orness* value. Specifically, we present the *orness* classification for the poverty measures of the Poverty Gap Ratio, the two popular indices introduced by Sen

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