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Dependence measures for perturbations of copulas

Jozef Komorník [a], Magda Komorníková [b,*], Jana Kalická [b]

[a] Faculty of Management, Comenius University, Odbojárov 10, P.O. BOX 95, 820 05 Bratislava, Slovakia
[b] Faculty of Civil Engineering, Slovak University of Technology, Radlinského 11, 810 05 Bratislava, Slovakia

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Abstract

In this paper (which is a substantially extended version of a conference paper from AGOP 2015 [10]), we investigate the effects of specific class of perturbations of bivariate copulas on several measures of dependence (Spearman's rho, Blomqvist's beta, Gini's gamma, Kendall's tau), and tail dependence along both diagonal sections. It is demonstrated that the influence of the perturbation parameter on the values of the first three of the above coefficients of dependence is linear, while on the last one it is quadratic. Interesting numerical analyses for several important classes of Archimedean copulas are presented. It is also demonstrated that the considered perturbations do not change the coefficients of tail dependencies along the main diagonal but linearly reduce their values along the second diagonal. An interesting possible application for analyzing dependencies along the second diagonal of copulas represent insurance data, where censoring introduces a negative dependence between the investigated components of the claims. As a by-product, we present a new class of perturbations of copulas that linearly reduce the more popular coefficients of tail dependencies along the main diagonal, while preserving their values along the second diagonal. Subsequently using suitable elements of both above classes of perturbations, any original copula can be transformed to a resulting one, having coefficients of tail dependencies along both diagonals linearly reduced (with any couple of preselected linear proportions from $[0, 1]$).

Keywords: Copula; Perturbation of copula; Tail dependence; Dependence measures

1. Introduction

Fitting of an appropriate copula to real data is one of the major tasks in application of copulas. For this purpose, a large buffer of potential copulas has been designed (mainly parametric families of copulas). Once we know approximately a copula C appropriate to the model of the observed data, we look for a minor perturbation of C which fits them better than C itself, compare [4]. We have applied this approach in our papers [11,13] where we used the classes of perturbed copulas introduced in [13] and extended in [11] to modeling relations between returns of investments in

* Corresponding author.
E-mail addresses: jozef.komornik@fm.uniba.sk (J. Komorník), magdalena.komornikova@stuba.sk (M. Komorníková), jana.kalicka@stuba.sk (J. Kalická).

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a selected class of REIT (Real Estate Investment Trust) indexes. We have concluded that the best models were quite often obtained in the form of perturbations with nonzero values of perturbation parameters.

There are several methods for fitting copulas to real data [3,5,8,14,15], and mostly the dependence parameters of the theoretical copulas only approximately copy the empirical dependence parameters. Then one can apply a perturbation of the considered theoretical copula to reach a consensus of a chosen dependence parameter between the theoretical and empirical case. This idea has inspired us to have a closer look to dependence parameters of perturbations of copulas, as well as to their tail dependencies.

The paper is organized as follows. The second section presents a brief overview of the theory of copulas. In the third section perturbations of bivariate copulas are discussed. The fourth section is devoted to an overview of the dependence measures of perturbed copulas and contains the main results of this paper dealing with tail dependencies. Finally, some concluding remarks are added.

2. Copulas

Copula represents a multivariate distribution that captures the dependence structure among random variables. It is a great tool for building flexible multivariate stochastic models. Copula offers the choice of an appropriate model for the dependence between random variables independently from the selection of marginal distributions. This concept was introduced in the late 50's and became popular in several fields beyond statistics and probability theory, such as finance, actuarial science, fuzzy set theory, hydrology, civil engineering, etc.

Definition 1. A function $C : [0,1]^2 \to [0,1]$ is called a (bivariate) copula whenever it is

i) 2-increasing, i.e., the volume over the rectangle $[u_1,u_2] \times [v_1,v_2]$

$$V_C\left([u_1,u_2] \times [v_1,v_2]\right) = \\ = C(u_1,v_1) + C(u_2,v_2) - C(u_1,v_2) - C(u_2,v_1) \geq 0$$

for all $0 \leq u_1 \leq u_2 \leq 1,\ 0 \leq v_1 \leq v_2 \leq 1$ (recall that $V_C\left([u_1,u_2] \times [v_1,v_2]\right)$ is the C-volume of the rectangle $[u_1,u_2] \times [v_1,v_2]$);

ii) grounded, i.e., $C(u,0) = C(0,v) = 0$ for all $u, v \in [0,1]$;

iii) it has a neutral element $e = 1$, i.e., $C(u,1) = u$ and $C(1,v) = v$ for all $u, v \in [0,1]$.

Recall that for a 2-dimensional random vector (X,Y) with a joint distribution function F_{XY} and continuous marginal distribution functions F_X, F_Y a copula C satisfying the relations $F_{XY}(x,y) = C(F_X(x), F_Y(y))$ is the distribution function of the random vector (U,V), where $U = F_X(X)$ and $V = F_Y(Y)$ have uniform distributions on $[0,1]$. For more details we recommend monographs Joe (1997) [8] and Nelsen (2006) [14].

We follow the approach of Patton (2006) [15] and consider a so-called *survival copula* derived from a given copula C corresponding to the couple (X,Y) by

$$\widehat{C}(u,v) = u + v - 1 + C(1-u, 1-v) \tag{1}$$

which is the copula corresponding to the couple $(-X,-Y)$.

Another natural transformations of the copula C are copulas LC and RC corresponding to the couples $(-X,Y)$ and $(X,-Y)$, respectively.

They have the form

$$LC(u,v) = v - C(1-u, v)$$

and

$$RC(u,v) = u - C(u, 1-v).$$

We will call the copulas LC and RC the *left* and the *right reflections* of the copula C, respectively (see e.g. [2]). Since the survival copula $\widehat{C}$ can be obtained in the form of the right reflection of the copula LC as well as the left