ARTICLE IN PRESS
FSS:7139

Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems ••• (••••) •••–•••

FUZZY sets and systems

www.elsevier.com/locate/fss

Constructing strict left (right)-disjunctive left (right) semi-uninorms and coimplications satisfying the order property [☆]

Zhudeng Wang [a,*], Yuan Wang [b], Meixia Niu [a,c], Xiaoying Hao [a,c]

[a] School of Mathematics and Statistics, Yancheng Teachers University, Jiangsu 224002, People's Republic of China
[b] College of Information Engineering, Yancheng Teachers University, Jiangsu 224002, People's Republic of China
[c] Department of Mathematics, Qinghai Normal University, Xining 810000, People's Republic of China

Received 21 December 2014; received in revised form 6 December 2016; accepted 7 December 2016

Abstract

In this paper, we further study the constructions of left (right) semi-uninorms and coimplications on a complete lattice. We firstly give out the formulas for calculating the upper and lower approximation strict left (right)-disjunctive left (right) semi-uninorms of a binary operation. Then, we lay out the formulas for calculating the upper and lower approximation coimplications, which satisfy the order property, of a binary operation. Finally, we investigate the relationships between the lower approximation strict left (right)-disjunctive left (right) arbitrary $\wedge$-distributive left (right) semi-uninorms and upper approximation right arbitrary $\vee$-distributive coimplications which satisfy the order property, and give some conditions such that the upper approximation strict left (right)-disjunctive left (right) semi-uninorms of a binary operation and lower approximation coimplication, which satisfies the order property, of the left (right) deresiduum of the binary operation satisfy the generalized dual modus ponens rule.

Keywords: Fuzzy connective; Left (right) semi-uninorm; Coimplication; Strict left (right)-disjunctive; Upper (lower) approximation; Order property

1. Introduction

Uninorms, introduced by Yager and Rybalov [28], and studied by Fodor et al. [7], are special aggregation operators that have proven useful in many fields like fuzzy logic, expert systems, neural networks, aggregation, and fuzzy system modeling (see [8,26,27]). This kind of operation is an important generalization of both t-norms and t-conorms and a special combination of t-norms and t-conorms (see [7]). But, there are real-life situations when truth functions cannot be associative or commutative. By throwing away the commutativity from the axioms of uninorms, Mas et al. introduced the concepts of left and right uninorms on [0, 1] in [13] and later on a finite chain in [14], and Wang and

☆ This work is supported by the Jiangsu Provincial Natural Science Foundation of China (BK20161313), Science Foundation of Yancheng Teachers University (16YCKLQ006) and National Natural Science Foundation of China (11571006).
* Corresponding author.
E-mail addresses: yctuwangzhudeng@163.com, zhudengwang2004@163.com (Z. Wang).

http://dx.doi.org/10.1016/j.fss.2016.12.006

Please cite this article in press as: Z. Wang et al., Constructing strict left (right)-disjunctive left (right) semi-uninorms and coimplications satisfying the order property, Fuzzy Sets Syst. (2016), http://dx.doi.org/10.1016/j.fss.2016.12.006

Fang [23,24] studied the left and right uninorms on a complete lattice. By removing the associativity and commutativity from the axioms of uninorms, Liu [12] introduced the concept of semi-uninorms, and Su et al. [20] discussed the notions of left and right semi-uninorms, on a complete lattice. On the other hand, it is well known that a uninorm (semi-uninorm) U is conjunctive or disjunctive whenever $U(0,1) = 0$ or 1, respectively. This fact allows us to use uninorms in defining fuzzy implications and coimplications (see [4,15–18]).

Constructing fuzzy connectives is an interesting topic. Recently, Jenei and Montagna [11] introduced several new types of constructions of left-continuous t-norms, Wang [21] laid bare the formulas for calculating the smallest pseudo-t-norm that is stronger than a binary operation and the largest implication that is weaker than a binary operation, Su et al. [20] studied the constructions of left and right semi-uninorms, and Su and Wang [19] investigated the constructions of implications and coimplications on a complete lattice. This paper is a continuation of [19,20]. In this paper, motivated by these works, we will further focus on this issue and investigate the constructions of strict left (right)-disjunctive left (right) semi-uninorms and coimplications, which satisfy the order property, on a complete lattice.

The paper is organized as follows. In Section 2, we briefly recall the concepts of left (right) semi-uninorms and coimplications on a complete lattice and illustrate these notions by means of a theorem and some examples. In Section 3, we give out the formulas for calculating the upper and lower approximation strict left (right)-disjunctive left (right) semi-uninorms of a binary operation. In Section 4, we lay out the formulas for calculating the upper and lower approximation coimplications, which satisfy the order property, of a binary operation. In Section 5, we investigate the relationships between the lower approximation strict left (right)-disjunctive left (right) arbitrary $\wedge$-distributive left (right) semi-uninorms and upper approximation right arbitrary $\vee$-distributive coimplications which satisfy the order property, and give some conditions such that the upper approximation strict left (right)-disjunctive left (right) semi-uninorms of a binary operation and lower approximation coimplication, which satisfies the order property, of the left (right) deresiduum of the binary operation satisfy the generalized dual modus ponens rule.

The knowledge about lattices required in this paper can be found in [1].

Throughout this paper, unless otherwise stated, L always represents any given complete lattice with maximal element 1 and minimal element 0; J stands for any index set.

2. Left (right) semi-uninorms and coimplications on a complete lattice

In this section, we briefly recall some necessary concepts about the left (right) semi-uninorms and coimplications on a complete lattice, and illustrate these notions by means of a theorem and some examples.

Definition 2.1 *(Su et al. [10,20])*. A binary operation U on L is called a left (right) semi-uninorm if it satisfies the following two conditions:

(U1) there exists a left (right) neutral element, i.e., an element $e_L \in L$ ($e_R \in L$) satisfying $U(e_L, x) = x$ ($U(x, e_R) = x$) for all $x \in L$,
(U2) U is non-decreasing in each variable.

If a left (right) semi-uninorm U on L is associative, then U is the left (right) uninorm in [23,24].

If a left (right) semi-uninorm U with the left (right) neutral element e_L (e_R) has a right (left) neutral element e_R (e_L), then $e_L = U(e_L, e_R) = e_R$. Let $e = e_L = e_R$. Here, U is the semi-uninorm in [12].

For a left (right) semi-uninorm U on L, U is said to be left-conjunctive (right-conjunctive) if $U(0,1) = 0$ ($U(1,0) = 0$). U is called conjunctive if both $U(0,1) = 0$ and $U(1,0) = 0$ since it satisfies the classical boundary conditions of AND. U is said to be left-disjunctive (right-disjunctive) if $U(1,0) = 1$ ($U(0,1) = 1$). We call U disjunctive if both $U(1,0) = 1$ and $U(0,1) = 1$ by a similar reason.

U is said to be strict left-disjunctive (strict right-disjunctive) if U is disjunctive and for any $x \in L$, $U(x,0) = 1 \Leftrightarrow x = 1$ ($U(0,x) = 1 \Leftrightarrow x = 1$).

Definition 2.2 *(De Baets [3], De Baets and Fodor [4], Fodor and Roubens [6])*. A coimplication C on L is a hybrid monotonous (with decreasing first and increasing second partial mappings) binary operation that satisfies the corner conditions $C(0,0) = C(1,1) = 0$ and $C(0,1) = 1$.

Please cite this article in press as: Z. Wang et al., Constructing strict left (right)-disjunctive left (right) semi-uninorms and coimplications satisfying the order property, Fuzzy Sets Syst. (2016), http://dx.doi.org/10.1016/j.fss.2016.12.006