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Several types of enriched (L, M)-fuzzy convergence spaces ☆

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Abstract

In this paper, several types of enriched (L, M)-fuzzy convergence spaces are introduced, including enriched (L, M)-fuzzy Kent convergence spaces, enriched (L, M)-fuzzy limit spaces, pretopological enriched (L, M)-fuzzy convergence spaces, topological enriched (L, M)-fuzzy convergence spaces and enriched (L, M)-fuzzy Choquet convergence spaces. These concepts generalize the concepts of Kent convergence spaces, of limit spaces, of pretopological convergence spaces, of topological convergence spaces and of Choquet convergence spaces in general topology to the setting of (L, M)-fuzzy topology. Also, their categorical properties and their mutual categorical relations are investigated.

Keywords: Topology; Category; Enriched (L, M)-fuzzy convergence structure; (L, M)-fuzzy filter; Bireflective (Bicoreflective) subcategory; Cartesian closed

1. Introduction

By “convergence”, we usually mean filter convergence or net convergence in topological spaces. However, in some respects, topological spaces are not the most suitable setting for studying the basic properties of convergence, especially the categorical properties. This leads to the axiomatic convergence structure theory, which is a branch of topology theory dealing with set-theoretic structures satisfying axioms similar to that usual filter convergence or net convergence fulfill. Axiomatic convergence structures, as a generalization of topological structures, mainly contain the following aspects:

- Relations with topological spaces in a categorical sense.
- Categorical properties, such as Cartesian closedness, extensionality and productivity of quotient maps.
- Topological properties, such as compactness and separation.

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In the famous book [31], Preuss developed a common framework for all of these theories. Besides generalized convergence spaces, other important types of convergence structures are introduced by various authors. In [18], Kent introduced a special kind of convergence structures, called Kent convergence structures. Afterwards, Kowalsky [19] and Fischer [5] independently proposed the concept of limit structures. Based on ultrafilters, Choquet [2] presented the notion of pseudotopological convergence structures (also named Choquet convergence structures). Using bireflectors and bicoreflectors in category theory, Preuss [31] described the relations among various types of convergence structures.

There are different approaches to generalize the notion of convergence structures to the lattice-valued setting. Based on prefilters, Lowen [24] introduced a kind of fuzzy convergence structures. Also, Min [25] and Lee [20] proposed the notion of fuzzy limit structures by means of prefilters, respectively. In [9], Höhle and Šostak proposed the concept of (stratified) L-filters. Based on this new kind of fuzzy filters, Jäger [10] introduced stratified L-fuzzy convergence spaces (which are called stratified L-generalized convergence spaces in [11]). There are so many following works to study this kind of fuzzy convergence structures [3,4,13–15,17,21–23,33]. In a completely different direction, Xu [32] introduced fuzzifying convergence structures by using ordinary filters and Yao [34] introduced L-fuzzifying convergence structures by using L-filters of ordinary subsets. In order to establish the lattice-valued convergence structure theory in the framework of (L, M)-fuzzy topology, Gähler [6] introduced monadic convergence structures and Jäger [16] proposed the concept of stratified LMN-convergence structures. In these two cases, the convergent points are all crisp points. Based on fuzzy points, Güloğu and Coker [7] introduced the concept of I-fuzzy ($I = [0, 1]$) convergence structures by means of I-filters. Later, Pang and Fang [26,27] used L-filters to define L-fuzzy Q-convergence structures, where L is a completely distributive lattice. Afterwards, Pang [28] introduced the concept of (L, M)-fuzzy convergence structures and established its categorical relations with (L, M)-fuzzy topologies. In [28], Pang showed the Cartesian closedness of the category of (L, M)-fuzzy convergence spaces, but failed to present the concrete form of function space structures. Adopting a different fuzzification approach, Pang [30] introduced enriched (L, M)-fuzzy convergence structures. With the enriched axiomatic condition, Pang constructed the corresponding function space structures to show the Cartesian closedness of the category of enriched (L, M)-fuzzy convergence spaces. As a continuation of [30], we will focus on several types of enriched (L, M)-fuzzy convergence spaces. Concretely, we will generalize the well-known categories of Kent convergence spaces, of limit spaces, of pretopological convergence spaces, of topological convergence spaces and of Choquet convergence spaces to the situation of enriched (L, M)-fuzzy convergence structures and investigate their categorical properties.

This paper is organized as follows. In Section 2, we recall some necessary concepts and notations. In Section 3, we introduce enriched (L, M)-fuzzy Kent convergence structures and study its relations with enriched (L, M)-fuzzy convergence structures in a categorical sense. In Section 4, we propose the concept of enriched (L, M)-fuzzy limit spaces and prove that the resulting category is a Cartesian closed topological category. In Section 5, we give some characterizations of pretopological enriched (L, M)-fuzzy convergence structures and study its relations with enriched (L, M)-fuzzy limit structures. In Section 6, we mainly investigate the categorical relations between pretopological enriched (L, M)-fuzzy convergence spaces and topological enriched (L, M)-fuzzy convergence spaces from a categorical aspect. In Section 7, we generalize Choquet convergence spaces to enriched (L, M)-fuzzy Choquet convergence spaces and study its relations with other kinds of enriched (L, M)-fuzzy convergence spaces.

2. Preliminaries

Throughout this paper, both L and M denote completely distributive lattices and $'$ is an order-reversing involution on L. The smallest element and the largest element in L (M) are denoted by $\bot_L$ ($\bot_M$) and $\top_L$ ($\top_M$), respectively. For $a, b \in L$, we say that a is wedge below b, in symbols $a \prec b$, if for every subset $D \subseteq L$, $\bigvee D \geqslant b$ implies $d \geqslant a$ for some $d \in D$. An element a in L is called coprime if $a \leqslant b \vee c$ implies $a \leqslant b$ or $a \leqslant c$. The set of nonzero coprime elements in L and M is denoted by $J(L)$ and $J(M)$, respectively. A complete lattice L is completely distributive if and only if $b = \bigvee\{a \in J(L) \mid a \prec b\}$ for each $b \in L$. An element a in L is called prime if $a \geqslant b \wedge c$ implies $a \geqslant b$ or $a \geqslant c$.

For a nonempty set X, L^X denotes the set of all L-subsets on X. L^X is also a complete lattice when it inherits the structure of the lattice L in a natural way, by defining $\vee$, $\wedge$ and $\leqslant$ pointwisely. The smallest element and the largest element in L^X are denoted by $\bot_L^X$ and $\top_L^X$, respectively. For each $x \in X$ and $a \in L$, the L-subset x_a, defined by $x_a(y) = a$ if $y = x$, and $x_a(y) = \bot_L$ if $y \neq x$, is called a fuzzy point. The set of nonzero coprime elements in

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