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An efficient method to factorize fuzzy attribute-oriented concept lattices

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Abstract

Factorization by similarity is a mathematical technique used in formal concept analysis with fuzzy attributes for reducing the complexity of different types of fuzzy concept lattices. In this paper we find the structure of the factor lattice of a fuzzy attribute-oriented concept lattice, namely the intervals representing the blocks of this lattice. We provide a procedure for generating the infimum and the supremum concepts of these intervals as fixpoints of a fuzzy closure operator. This theoretical result allows to develop a more efficient algorithm for building the factor lattice of the fuzzy attribute-oriented concept lattice. A comparison between our approach and the existing algorithms is presented.

Keywords: Similarity relation; fuzzy attribute-oriented concept lattice

1. Introduction

Over the last decades, researchers have faced new challenging issues in order to deal with the huge amount of information available in databases. Developed as a branch of applied lattice theory, Formal Concept Analysis (FCA) is a mathematical framework which provides techniques to extract, organize and represent the relevant data hidden in large data tables. In the classical setting of FCA, originally proposed by R. Wille in [38], the information is described as an ordered triple called formal context, consisting of a set of objects, a set of attributes (or properties) of objects, and a binary relation between the objects and their attributes. Using a Galois connection, the FCA methods allow to process the data generating a collection of concepts which are complete clusters of objects having an attribute conjunctive description. The set of these classes of objects equipped with the subconcept–superconcept partial order forms a complete lattice called concept (or Galois) lattice.

In their traditional setting, FCA formulas and theoretical results are based on classical logic. By considering contexts with attributes taking values in a set of truth degrees endowed with appropriate operations, researchers have

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developed several extensions of Wille's approach to the fuzzy setting. Thus, conceptual scaling was proposed in [20] as a method to deal with the so-called multi-valued contexts, while fuzzy logic was first integrated into FCA in [11]. However, considering residuated lattices as structures of truth degrees, it was developed a feasible approach for FCA with graded attributes in [3] and, independently, in [31]. The authors used an antitone Galois connection to define the notions of formal fuzzy concept and fuzzy concept lattice. Using two different types of isotone derivation operators and non-commutative logic, the classical approach presented in [16] is extended in [21] by introducing two types of fuzzy concept lattices, namely the fuzzy object-oriented and the fuzzy attribute-oriented concept lattices. In [9], FCA with fuzzy attributes is extended to the case when the partial order on the set of fuzzy sets is replaced with a fuzzy order. In the literature, we can find even further generalizations of the notion of concept lattice [18,24,26,30].

Both the accessibility and the generality of FCA make the theory suitable to be applied in various fields. Thus, FCA has provided a mathematical basis for conceptual knowledge processing [39], data mining and design in software engineering [35], gene expression data analysis [22], hierarchical classification of web search results [14,28], analysis of software code [33] and information retrieval [12].

Generating the concept lattice and its associated Hasse diagram is one of the major issues in FCA. This process can be computationally expensive, particularly when dealing with real world data tables. Various techniques have been developed over the last years in order to reduce the size of the Galois lattice. The research efforts were first directed to obtain more efficient algorithms for generating and representing the set of concepts [19,25,27,34]. A method to control the size of the concept lattice through two unary functions (called hedges) defined on the residuated lattice was proposed in [7]. A method to simplify the information provided by a fuzzy biconcept lattice taking into account the membership degree of objects or attributes was proposed in [1]. A different approach to obtain a less complex procedure is to reduce the size of the attribute set. Thus, some authors have recently studied the attribute reduction effect for fuzzy oriented concept lattices [37,29].

In order to reduce the complexity of the conceptual structure, methods to decompose and to structure the lattice into smaller parts have been also developed. FCA researchers have been investigated the conditions allowing the Galois lattice to be factorized. Thus, as a general framework, they used similarity relations to measure the degree of indistinguishability between concepts, and so to group the similar classes of objects. In [4] it is used a fuzzy similarity between concepts to factorize the fuzzy concept lattice, and so to generate the factor lattice. These ideas were extended in [10], where it is provided a general method for factorizing systems of fuzzy sets by similarity. In [17] it is proposed a similarity for semantic web by following an information content approach, and accordingly the similarity of concept intents is computed independently of the related extents. In [23] it is showed that the factor lattice of a fuzzy concept lattice can be computed from a special kind of super-relations of the incidence relation called fuzzy block relations. A faster algorithm for computing the factor lattice of an antitone fuzzy concept lattice was developed in [5]. Based on a fuzzy similarity relation, we defined in [13] a compatible tolerance relation on the set of fuzzy attribute-oriented concepts which is used to factorize the corresponding fuzzy concept lattice, and so to reduce the complexity of the conceptual structure.

The aim of this paper is to develop an efficient method to generate the factor lattice of a fuzzy attribute-oriented concept lattice. Following the results of [13], we present here an algorithm to build the factor lattice. Since such a procedure is usually costly, we investigate a simpler way to build the blocks, and so the factor lattice. In order to get a faster algorithm for computing the reduced conceptual structure, we closely analyze the structure of the factor lattice in the isotone case. Thus, we find the structure of the blocks which is different than the one corresponding to the case of antitone fuzzy Galois lattices [5]. Then we show that the blocks computation is reduced to the problem of finding the fixpoints of a fuzzy closure operator. Using this result, we develop an efficient method to compute the blocks of the resulted factor lattice directly from the input data. A set of experiments on large real databases which proves the efficiency of the method concludes our work.

The remainder of the paper is organized as follows. We briefly overview the basic notions and results of FCA with fuzzy attributes in Section 2. In Section 3 we first present the main issues involved in reducing the size of fuzzy attribute-oriented Galois lattices by similarity which are particularly connected with our approach. Then we find the structure of the factor lattice obtained by factorizing a fuzzy attribute-oriented Galois lattice via a compatible tolerance relation. Finally, we provide a theoretical method which allows to develop a more efficient algorithm for generating the blocks of the factor lattice. Section 4 presents some experimental results in order to evaluate the efficiency of the presented method.

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