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Finite-time stability of a class of fuzzy cellular neural networks with multi-proportional delays ☆

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Abstract

This paper is concerned with finite-time stability for a class of fuzzy cellular neural networks with multi-proportional delays. Based on the finite-time stability theory, we establish a novel result to ensure the finite-time stability of the addressed system by employing the differential inequality techniques. We also use numerical simulations to demonstrate our theoretical result.

Keywords: Fuzzy cellular neural networks; Finite-time stability; Proportional delay

1. Introduction

As pointed out in [1–6], in the implementation of neural networks (such as in the process of moving images), time delays unavoidably exist in the neural processing and signal transmission due to the finite switching speed of neurons and amplifiers. This will affect the stability of the neural system and may lead to some complex dynamic behavior, such as instability, chaos, oscillation or some other performance of the neural network. In particular, time delays may be proportional in theory, that is to say, the proportional delay function $\tau(t) = t - qt$ is a monotonically increasing function with the increase of time $t > 0$, where q is a constant and satisfies $0 < q < 1$. In fact, the proportional delay is one of the many objective-existent delay types such that the proportional delay is usually required in the web quality of service routing decisions. This is because it is convenient to control the network's run time according to the network allowed delays [7]. Moreover, the systems with proportional delays have many interesting applications, and there have been extensive results on the dynamic behavior of cellular neural networks (CNNs) with proportional delays. A summary of the results of CNNs with proportional delays are given, with the asymptotic stability and dissipativity [8–13]. Furthermore, exponential stability analysis was considered for CNNs with proportional delays [14–16], and the exponential synchronization was investigated [17].

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In recent years, there has been an interest in the analysis of finite-time stability (FTS) behavior for time-delay systems (see, for example, [18–22] and the references therein). It is worth mentioning here that, FTS and Lyapunov asymptotic stability (LAS) are different concepts, that is to say, a system may be finite-time stable but not Lyapunov asymptotically stable and vice versa [23]. Furthermore, FTS is a useful concept to study in many practical systems in the vivid world [24]. Therefore, by introducing a novel constructive approach, Le Van Hien and Doan Thai Son [25] derived some new explicit conditions in terms of matrix inequalities ensuring the FTS of CNNs with multi-proportional delays.

On the other hand, one special group of fundamental neural networks, fuzzy cellular neural networks (FCNNs), which integrate fuzzy logic into the structure of a traditional CNN and maintain the local connectedness among cells, has been introduced by Yang et al. [26,27]. Unlike previous CNN structures, FCNN has fuzzy logic between its template and input and/or output besides the "sum of product" operation. Meanwhile, many studies have revealed that FCNN is a useful paradigm for image processing problems, which is a cornerstone in image processing and pattern recognition. Therefore, it is of great importance to analyze the dynamical behaviors of FCNNs both in theory and applications [29–33]. However, to the best of the authors' knowledge, few authors have handled the FTS of FCNNs with proportional delays. Consequently, this paper is devoted to analyzing the FTS of the following FCNNs with multiple proportional delays:

$$\begin{cases} \dot{x}_i(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t) f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)u_j(t) + I_i(t) \\ \qquad + \bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(x_j(q_{ij}t)) + \bigvee_{j=1}^{n} \beta_{ij}(t)g_j(x_j(q_{ij}t)) \\ \qquad + \bigwedge_{j=1}^{n} T_{ij}(t)u_j(t) + \bigvee_{j=1}^{n} H_{ij}(t)u_j(t),\ t > 0, \\ x_i(0) = x_i^0,\ i \in J = \{1, 2, \cdots, n\}, \end{cases} \tag{1.1}$$

where $\alpha_{ij}(t), \beta_{ij}(t), T_{ij}(t)$ and $H_{ij}(t)$ are the elements of the fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feedforward MIN template and fuzzy feedforward MAX template, respectively (usually, $a \bigwedge b = \min\{a, b\}$, $a \bigvee b = \max\{a, b\}$, see [26,27]); $a_{ij}(t)$ and $b_{ij}(t)$ are the elements of feedback template and feedforward template; $\bigwedge, \bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively; $x_i(t), u_i(t)$ and $I_i(t)$ denote the state, input and bias of the ith neuron, respectively; $c_i(t)$ represents the rates with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs; $f_i(\cdot)$ and $g_i(\cdot)$ denote the nonlinear activation functions; $q_{ij}, i, j \in J = \{1, 2, \ldots, n\}$ are proportional delay factors and satisfy $0 < q_{ij} \le 1$, and $q_{ij}t = t - (1 - q_{ij})t$, in which $(1 - q_{ij})t$ is the transmission delay function, and $(1 - q_{ij})t \to \infty$ as $q_{ij} \ne 1$, $t \to \infty$; x_i^0 is the initial value of $x_i(t)$ at time $t = 0$, and $i \in J$.

The main contributions of this paper can be listed as follows: (i) Based on the idea of the Lyapunov functional method and differential inequality techniques, some novel finite-time criteria are proposed for FCNNs with multiple proportional delays. (ii) The sufficient conditions in our main results can be transformed into the linear matrix inequalities, which can be easily checked via the Matlab LMI toolbox.

The remainder of the paper is organized as follows. In Section 2, we will establish a set of sufficient conditions which ensure the finite-time stability of (1.1). Here, we also study the exponential generalized synchronization of all solutions. In Section 3, an example with its computer simulations is given to show the correctness of the theoretical results. We end this paper with a brief conclusion in Section 4.

2. Main results

For convenience, we let $\mathbb{R}^n$ denote the n-dimensional vector space endowed with the norm $\|x\|_\infty = \max_{i \in J} |x_i|$ for $x = (x_i) \in \mathbb{R}^n$. The set of real $m \times n$-matrices is denoted by $\mathbb{R}^{m \times n}$. For $X = (x_{ij}) \in \mathbb{R}^{m \times n}$, we let $|X| = (|x_{ij}|)$. Comparisons between vectors will be understood component-wise. Specifically, for $u = (u_i)$ and $v = (v_i)$ in $\mathbb{R}^n$, write $u \ge v$ $(u \le v)$ if $u_i \ge v_i$ $(u_i \le v_i)$ for all $i \in J$ and $u \succ v$ $(u \prec v)$ if $u_i > v_i$ $(u_i < v_i)$ for all $i \in J$. For a given vector $\xi \in \mathbb{R}^n, \xi \succ 0$, we denote $\xi^u = \max_{i \in J} \xi_i$ and $\xi_l = \min_{i \in J} \xi_i$.

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