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Application of prediction models using fuzzy sets: A Bayesian inspired approach

Felipo Bacani [b,*], Laécio C. de Barros [a]

[a] Department of Applied Mathematics, State University of Campinas (IMECC – UNICAMP), 13083-970, Campinas–SP, Brazil
[b] Department of Exact and Applied Sciences, Federal University of Ouro Preto (DECEA – UFOP), 35931-008, João Monlevade–MG, Brazil

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Abstract

A fuzzy inference framework based on fuzzy relations is developed, adapted and applied to temperature and humidity measurements from a specific coffee crop site in Brazil. This framework consists of fuzzy relations over possibility distributions, resulting in a model analogous to a Bayesian inference process. The application of this fuzzy model to a data set of experimental measurements and its correspondent forecasts of temperature and humidity resulted in a set of revised forecasts, that incorporate information from a historical record of the problem. Each set of revised forecasts was compared with the correspondent set of experimental data using two different statistical measures, MAPE (Mean Absolute Percentage Error) and Willmott's D. This comparison showed that the sets of forecasts revised by the fuzzy model exhibited better results than the original forecasts on both statistical measures for more than two thirds of the evaluated cases.

Keywords: Possibility theory; Fuzzy relations; Fuzzy inference systems

1. Introduction

This work presents an inference method with a Bayesian interpretation using fuzzy relations [38] and its possible applications. The method was applied to improve forecasts of temperature and relative humidity of specific coffee crop sites in Brazil [3]. From the theoretical point of view, the approach is similar to the one used in [5,33]. A comparison between the results of the proposed methodology and a classical Bayesian model will be made in future works.

Research looking for equivalent concepts in fuzzy theory and probability theories started back in the 70's, initiated by Zadeh [44] himself. He defined analogous concepts of independent and non-interactive random variables,[1] obtaining expressions for a conditional fuzzy set from the marginal and joint ones, arriving at more complicated expressions

* Corresponding author.
E-mail addresses: fbacani@decea.ufop.br (F. Bacani), laeciocb@ime.unicamp.br (L.C. de Barros).

[1] Independent and non-interactive random variables are equivalent concepts in probability theory, but is not true for Zadeh's fuzzy case. This is due to Zadeh's replacement of the product operator for the minimum in the equations from the probability theory.

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than the probabilistic case due to the use of a minimum operator instead of a product. We found out that one development of Zadeh's ideas was made by Nguyen [37], who proposed a particular normalization that guarantees the consistency of the non-interactivity concept defined by Zadeh. Nguyen goes on to obtain expressions for the "conditional" fuzzy set. Hisdal [23] takes the opposite direction to that of Zadeh and Nguyen, obtaining the joint distribution from the conditional and marginal ones, and managed to (in a sense) generalize the expressions obtained by Zadeh. In the 80's, Bouchon [8] provided an important reference for the approach that is adopted here, as it was apparently the first work that treated the subject as a fuzzy relational equation problem, which was the direction taken by Lapointe and Bobée [33]. Similar efforts were summarized in references [19, pp. 370–373], and [20] (Section 6.7).

1.1. *Forecast processing and the Possibilistic Processor of Forecasts (PPF) framework*

Forecast processing problems have been thoroughly studied under a probabilistic framework, where a solution based on Bayes formula has been proposed by Krzysztofowicz [28–32], in the particular context of hydrology and management of water resources. The general framework of forecast processing applications consists of the following. Suppose a decision-maker has to make a choice based on variable X. In order to make this choice, a specialist gives him/her a (imperfect) forecast Y of X and he/she has at hand an historical record of observed values of X and its corresponding forecasts Y. The decision-maker has to combine the forecast received with the historical information, taking into account what happened with previous forecasts in the same problem (specialists skill). In the probabilistic case, forecast skill is modeled by the likelihood function $f(y\,|\,x)$. The likelihood function and the prior probability density distribution of X, $h_0(x)$, are determined by the historical data.

Krzysztofowicz called his probabilistic framework *Bayesian Processor of Forecasts* (BPF) and, in reference to BPF, the fuzzy framework used in this work was called *Possibilistic Processor of Forecasts* (PPF) [33]. The PPF framework has the advantage of dealing with analytical expressions, instead of the approximate nature of the solutions that arises in the practical Bayesian context. The proposed fuzzy inference framework is made by obtaining solutions from a fuzzy relations problem that contains information from historical data. The analogy with the Bayesian process is made by identifying terms analogous to a prior distribution, a likelihood function and a posterior distribution on the fuzzy relations problem described.

2. Fuzzy inference framework

2.1. *Preliminaries*

2.1.1. *Bayesian inference*

In the forecasting context, the use of the Bayesian methodology is frequently used when historic information about the problem is scarce or useless [36, p. 241]. Let Y be a random variable with density function f, characterized by an unknown parameter X. Its density is denoted as $f(y\,|\,x) \equiv f(Y=y\,|\,X=x)$ to indicate the dependence on the value of X. The probability density function of the parameter X is denoted as $h_0(x) \equiv h_0(X=x)$ and is termed the *prior distribution*. Prior distributions expresses the subjective information ("belief degree") about the value of X.

In a forecast situation, suppose that an initial estimate of X is given as *prior distribution* $h_0(x)$, and subsequent information about the situation is obtained through a random variable Y, whose *likelihood function* $f(y\,|\,x)$ depends on x. The new estimate of X is obtained in the form of a revised distribution, $h_1(x\,|\,y) \equiv h_1(X=x\,|\,Y=y)$, known as the *posterior distribution*. If both random variables X and Y are continuous, an expression for the posterior distribution is given by Bayes' Theorem:

$$h_1(x\,|\,y) = \frac{h_0(x)\, f(y\,|\,x)}{\int_x h_0(x)\, f(y\,|\,x)\, dx}. \tag{2.1}$$

The posterior distribution $h_1(x\,|\,y)$ can be seen as a *trade-off* between information about the likelihood $f(y\,|\,x)$, which represents the evidence of the data by itself, and the prior distribution $h_0(x)$, that represents information about the random variable X before any evidence has been taken into account.

2.1.2. *Fuzzy relations and the Compositional Rule of Inference*

T-norms Triangular norms (t-norms) are used to define fuzzy set aggregation operators. A t-norm is a function $T : [0,1] \times [0,1] \rightarrow [0,1]$, $T(a,b) = a\,T\,b$, that satisfies $\forall a, a', b, b', c \in [0,1]$:

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