ARTICLE IN PRESS
JID:FSS AID:7111 /FLA [m3SC+; v1.237; Prn:27/10/2016; 15:30] P.1 (1-30)
Available online at www.sciencedirect.com
ScienceDirect
Fuzzy Sets and Systems ••• (••••) •••–•••
FUZZY sets and systems
www.elsevier.com/locate/fss

On some types of fuzzy covering-based rough sets

Bin Yang [a,b], Bao Qing Hu [a,b,*]

[a] School of Mathematics and Statistics, Wuhan University, Wuhan 430072, PR China
[b] Computational Science Hubei Key Laboratory, Wuhan University, Wuhan 430072, PR China

Received 21 November 2015; received in revised form 25 March 2016; accepted 18 October 2016

Abstract

Fuzzy coverings are a natural extension of the coverings by replacing crisp sets with fuzzy sets. Recently, an excellent introduction to the definition of a fuzzy β-covering is due to Ma and two fuzzy covering-based rough set models are presented. In this paper, by introducing some new definitions of fuzzy β-covering approximation spaces, the properties of a fuzzy β-covering approximation space and Ma's fuzzy covering-based rough set models are studied. Furthermore, three new types of fuzzy covering-based rough set models as generalizations of Ma's models are first proposed in this paper. First, some properties of fuzzy β-covering and its fuzzy β-neighborhood family are proposed. We present a necessary and sufficient condition for fuzzy β-neighborhood family induced by a fuzzy β-covering to be equal to the fuzzy β-covering itself. Then we study the characterizations of Ma's fuzzy covering-based rough set models and give a necessary and sufficient condition for two fuzzy β-coverings to generate the same fuzzy covering lower approximation or the same fuzzy covering upper approximation. Finally, this paper proposes three new types of fuzzy covering-based rough set models by introducing a new notion of a fuzzy complementary β-neighborhood.

Keywords: Fuzzy sets; Fuzzy β-covering; Fuzzy covering-based rough sets; Fuzzy β-neighborhood; Fuzzy complementary β-neighborhood

1. Introduction

Rough set theory (RST) was originally proposed by Pawlak [31,33] in 1982 as a useful mathematical tool for dealing with the vagueness and granularity in information systems and data analysis. This theory can approximately characterize an arbitrary subset of a universe by using two definable subsets called lower and upper approximations [3]. In Pawlak's rough set model, the relationships of objects were built on equivalence relations. All equivalence classes form a partition of a universe of discourse. However, an equivalence relation imposes restrictions and limitations on many applications [6,16,19,23,50]. Hence, many extensions have been made in recent years by replacing equivalence relation or partition by notions such as binary relations [11,22,41,42,56,57], neighborhood systems and Boolean algebras [1,51,58], and coverings of the universe of discourse [2,34,35]. Based on the notion of covering, Pomykala [34,35] obtained two pairs of dual approximation operators. Yao [58] further examined these approximation operators

* Corresponding author at: School of Mathematics and Statistics, Wuhan University, Wuhan 430072, PR China.
E-mail addresses: binyang0906@whu.edu.cn (B. Yang), bqhu@whu.edu.cn (B.Q. Hu).

http://dx.doi.org/10.1016/j.fss.2016.10.009
0165-0114/

Please cite this article in press as: B. Yang, B.Q. Hu, On some types of fuzzy covering-based rough sets, Fuzzy Sets Syst. (2016), http://dx.doi.org/10.1016/j.fss.2016.10.009

by the concepts of neighborhood and granularity. Such undertaking has stimulated more research in this area [4,24,25,60,63–67]. Over the past 30 years, RST has indeed become a topic of great interest to researchers and has been applied to many domains. This success is due in part to the following three aspects of the theory:

(1) Only the facts hidden in data are analyzed.
(2) No additional information about the data is required such as thresholds or expert knowledge.
(3) A minimal knowledge representation can be attained.

However, RST is designed to process qualitative (discrete) data, and it faces great limitations when dealing with real-valued data sets since the values of the attributes in the databases could be both symbolic and real-valued [17]. Fuzzy set theory (FST) [62], is very useful to overcome these limitations, as it can deal effectively with vague concepts and graded indiscernibility. Nowadays, rough set theory and fuzzy set theory are the two main tools used to process uncertainty and incomplete information in the information systems. The two theories are related but distinct and complementary [32,59]. In the past twenty years, research on the connection between rough sets and fuzzy sets has attracted much attention. Intentions on combining rough set theory and fuzzy set theory can be found in different mathematical fields [30,49,59]. Dubois and Prade first proposed the concept of fuzzy rough sets [9], which combined these two theories and influenced numerous authors who used different fuzzy logical connectives and fuzzy relations to define fuzzy rough set models. Then more scholars generalized the fuzzy rough sets by using various methods [12–15,7,27–29,36,37,46,52,53,61]. The most common fuzzy rough set models are obtained by replacing the crisp binary relations and the crisp subsets with the fuzzy relations and the fuzzy subsets on the universe respectively.

As well as RST, some researchers have tried to generalize the fuzzy rough set based on fuzzy relations by using the concept of a fuzzy covering. De Cock et al. [5] defined fuzzy rough sets based on the R-foresets of all objects in a universe of discourse with respect to (w.r.t.) a fuzzy binary relation. When R is a fuzzy serial relation, the family of all R-foresets forms a fuzzy covering of the universe of discourse. Analogously, Deng [8] examined the issue with fuzzy relations induced by a fuzzy covering. Li and Ma [20], on the other hand, constructed two pairs of fuzzy rough approximation operators based on fuzzy coverings, the standard min operator $\mathscr{T}_M$, and the Kleene-Dienes implicator $\mathscr{I}_{KD}$. It should be noted that fuzzy coverings in the models proposed by Deng [8] and De Cock et al. [5] are induced from fuzzy relations. So, they are not fuzzy coverings in the general sense. Although fuzzy coverings are used by Li and Ma [20] in their models, they only employed two special logical operators i.e. the standard min operator and the Kleene-Dienes implicator. Thus, it is necessary to construct more general fuzzy rough set models based on fuzzy coverings. Recently, an excellent introduction to the topic of fuzzy covering-based rough sets is due to some scholars [10,21,39,45,55], which can be regarded as a bridge linking covering-based rough set theory and fuzzy rough set theory. The original definition of a fuzzy covering is defined as follows (see [10,21]).

Let U be an arbitrary universal set, and $\mathcal{F}(U)$ be the fuzzy power set of U. We call $\widehat{\mathbf{C}} = \{C_1, C_2, \ldots, C_m\}$, with $C_i \in \mathcal{F}(U)$ $(i = 1, 2, \ldots, m)$, a fuzzy covering of U, if $(\bigcup_{i=1}^{m} C_i)(x) = 1$ for each $x \in U$.

In fact, there exist some limits of this definition in the practical applications. To overcome these limits, Ma [26] generalized the fuzzy covering to fuzzy β-covering by replacing 1 with a parameter β $(0 < \beta \le 1)$. Furthermore, Ma defined two new types of fuzzy covering-based rough set models by introducing the new concept of a fuzzy β-neighborhood. Moreover, properties of the models and their relationships with other rough sets are discussed. In this paper, we study the properties of a fuzzy β-covering approximation space and Ma's fuzzy covering-based rough set models by introducing some new definitions of fuzzy β-covering approximation spaces. This paper obtains a necessary and sufficient condition for fuzzy β-neighborhood family induced by a fuzzy β-covering to be equal to the fuzzy β-covering itself. In [24], the concept of a complementary neighborhood and some types of covering-based rough set models are proposed from the viewpoint of application. Similarly, by introducing the notion of a fuzzy complementary β-neighborhood, three new types of fuzzy covering-based rough set models are defined in this paper. The properties of the models and their relationships with Ma's models are discussed.

The remainder of this paper is organized as follows. In Section 2, some preliminary definitions used in this paper are introduced. In Section 3, some properties of a fuzzy β-covering and its fuzzy β-neighborhood family are proposed. In Section 4, the characterizations of Ma's fuzzy covering-based rough set models are investigated. In Section 5, three new types of fuzzy covering-based rough set models are defined. Section 6 concludes this paper.