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Multi-granulation fuzzy preference relation rough set for ordinal decision system

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Abstract

Preference analysis is a class of important issues in multi-criteria ordinal decision making. Rough set is an effective approach to handle preference analysis. In order to solve the multi-criteria preference analysis problem, this work improves the fuzzy preference relation rough set model with additive consistent fuzzy preference relation, and expands it to multi-granulation case. Cost is also an important issue in decision analysis. Taking the cost into consideration, we also expand the model to cost sensitive multi-granulation fuzzy preference relation rough set. Some theorems are represented, and the classification and sample condensation algorithms based on our model are investigated. Some experiments are complete and the experimental results show that our model and algorithms are effective for preference decision making of ordinal decision system.

Keywords: Fuzzy rough set; Fuzzy preference relation; Multi-granulation; Classification; Ordinal decision system

1. Introduction

The concept of rough sets was originally proposed by Pawlak [1]. It is an extension of the set theory for the study of intelligent systems characterized by insufficient and incomplete information. A key notion in Pawlak's rough set model is equivalence relation. Given an indiscernibility relation, we then obtain a partition on the universe immediately [2]. By replacing crisp binary relations with fuzzy relations in the universe, Dubois and Prade proposed the concept of fuzzy rough set [3,4]. The Pawlak's rough set and fuzzy rough set are effective mathematical approach to handle imprecision, vagueness, and uncertainty in data analysis [5], and is used widely in data mining [6], machine learning [7], feature selection [8], decision support [9] and so on.

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However, there exist some other situations called ordinal decision making, where the decision maker is usually asked to give his/her preferences over alternatives, such as credit approval, stock risk estimation, and teaching evaluation [10]. Preference relations are very useful in expressing decision maker's preference information in ordinal decision problems. Fuzzy preference relation is first proposed by Orlovsky (1978) to represent an expert's opinion about a set of alternatives. The fuzzy preference relation not only can reflect that one alternative is prior to another alternative, but also can show the preference degree. The Pawlak's rough set model and fuzzy rough set are not able to receive and extract the information of ordinal structure and cannot be used to analyze the information with preference relations. Pawlak discussed this problem in [11]. Greco et al. proposed a novel rough set model for preference analysis and constructed dominance relation based on the decision preference [12–14]. S. Greco et al. proposed a rough approximation method of preference relation by dominance relations in [15,16]. Q.H. Hu et al. proposed a type of fuzzy preference relation rough sets model in [17].

From the viewpoint of the Granular Computing, by using a binary relation, objects are granulated into a set of information granules, called granule structures. Presently, three important multi-granulation rough sets have been proposed, they are optimistic, pessimistic and β-multi-granulation approaches. The Pawlak's rough set and fuzzy rough set are constructed based on one and only one partition and are regarded as single granulation rough set approach which cannot always be satisfying in many practical issues. Qian and Liang et al. proposed the concept of the multi-granulation rough set [18–21]. Xibei Yang et al. introduced test cost sensitive multi-granulation rough set by taking the test cost into consideration [2]. And in this work, we combine multi-granulation rough set with fuzzy preference relation and introduce a cost sensitive multi-granulation fuzzy preference relation rough set model.

Based on this idea, the contribution of this paper includes: (1) this paper introduces an additive consistent fuzzy preference relation and improves the fuzzy preference relation rough set model proposed by Q.H. Hu et al.; (2) proposes an additive consistent multi-granulation fuzzy preference relation rough set model in order to solve the multi-criteria preference analysis problem; (3) taking the cost into consideration, we also expand the model to cost sensitive multi-granulation fuzzy preference relation rough set; (4) furthermore, the classification and sample condensation algorithms are investigated and some experiments are completed. The experimental results show that for strict monotonic ordinal decision system, we can obtain very good classification quality with the approach proposed in this work, and for not strict monotonic ordinal decision system, we can improve the classification quality by deleting exceptional samples.

The paper is organized as follows: in Section 2, some basic concepts about rough set, fuzzy roughs set, multi-granulation rough set and fuzzy preference relation rough set are briefly represented. In Section 3, multi-granulation fuzzy preference relation rough set model and cost sensitive multi-granulation fuzzy preference relation rough set are proposed. A series of theorems are discussed. Section 4 represents the application of our model to classification and sample condensation. Some experiments are reported in Section 5. Finally, Section 6 concludes the paper.

2. Preliminary knowledge

In this section, we will review some basic concepts such as rough set, fuzzy rough set, multi-granulation rough set and fuzzy preference relation rough set, which have been addressed in [22–31].

2.1. Rough set and fuzzy rough set

Let U denote a finite and nonempty set called the universe, $R \in U \times U$ is an equivalence relation on U, (U, R) is called an approximation space. The relation R decomposes the universe U into some disjoint classes denoted by U/R

$$U/R = \{X_1, X_2, X_3, \ldots, X_N\}$$

where X_i is an equivalence relation in term of R, $i = 1, 2, \ldots, N$. If two elements x, y in U belong to the same equivalence class, we say x and y are indiscernible. $\forall x \in U$, $[x]_R$ is used to denote the equivalence class in term of R, which contains x.

$$[x]_R = \left\{y \in U : a(x) = a(y), \forall a \in R\right\}$$

Given an arbitrary set $X \subseteq U$, R is an equivalence relation on U, the lower approximation is the greatest definable set contained in X, and the upper approximation is the least definable set containing X. They can be computed by the following equivalent formulas.