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Short Communication

Ranking methods for fuzzy numbers: The solution to Brunelli and Mezei's conjecture

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Abstract

In this study, a solution to the conjecture presented by M. Brunelli and J. Mezei is proposed. As per the subject solution, it is demonstrated that two ranking methods proposed by Yager and Nakamura induce the same ranking for all fuzzy numbers.

Keywords: Fuzzy numbers; Ranking methods; Ordering of fuzzy quantities

1. Introduction

The problem of ordering fuzzy quantities has been addressed by many researchers over the years, and a large number of ranking methods have been proposed. In [3], M. Brunelli and J. Mezei tried to study by means of numerical simulation to what extent results obtained by several ranking methods are similar in practice. These researchers used the Pearson correlation coefficient, which measures linear dependence between two variables to study the correlation between pairs of ranking approaches. They essentially determined that there are some very similar methods at hand as well as some outliers, which led them to the ultimate conclusion that the value of the Spearman index between Y_2 and $N^{0.5}$, which are proposed by Yager [12–14] and Nakamura [8], equals 1. The authors proposed a conjecture, based on their experimental results.

Conjecture. *Methods Y_2 and $N^{0.5}$ induce the same ranking for all the trapezoidal fuzzy numbers.*

To prove this conjecture, we introduce some definitions and notations for fuzzy sets and ranking methods. Subsequently we prove that the conjecture is true for all fuzzy numbers in Section 3.

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2. Preliminary

Given a universe X, a fuzzy set A on X is a set $\{(x, \mu_A(x)) \,|\, x \in X\}$ where $\mu_A(x)$ is the membership degree of $x \in A$. Given a universe X and a fuzzy set A, an α-cut A_α is the subset defined as $\{x \in X | \mu_A(x) \geq \alpha\}$ with $\alpha \in (0, 1]$. The support of a fuzzy set is defined as $suppA = \text{cl}\{x \in X | \mu_A(x) > 0\}$ and A_0 is defined by $A_0 = suppA$. A fuzzy set defined on the real line is fuzzy convex if all its α-cuts are convex. A fuzzy set is normal if $\sup \mu_A(x) = 1$. A fuzzy number is a normal and convex fuzzy set on the real line, i.e., $X = \mathbb{R}$. By definition all the α-cuts of fuzzy numbers are intervals and therefore they can be conveniently defined by means of their endpoints, $a_\alpha^- = \inf A_\alpha$ and $a_\alpha^+ = \sup A_\alpha$.

2.1. Ranking methods

Given a ranking method, the notations $A \succ B$, $A \sim B$ and $A \succeq B$ mean that A has a higher ranking than B, the same ranking as B and at least the same ranking as B, respectively.

2.1.1. Yager's approach [12–14]

Yager proposed a ranking method for fuzzy quantities in the unit interval.

$$Y_2(A) = \int_0^{hgt(A)} M(A_\alpha)\, d\alpha,$$

where $hgt(A) = \sup_{x \in suppA} \mu_A(x)$ is the height of A and M is the mean value operator, can be used to rank fuzzy numbers with arbitrary support. In this case $hgt(A) = 1$ and $M(A_\alpha) = \frac{a_\alpha^- + a_\alpha^+}{2}$.

2.1.2. Nakamura's approach [8]

Nakamura defined the following parametric method based on the fuzzy relation

$$P_{N^\lambda}(A, B) = \frac{\lambda d_H\left(\underline{A}, \widetilde{\min}\left(\underline{A}, \underline{B}\right)\right) + (1-\lambda)\left(d_H\left(\overline{A}, \widetilde{\min}\left(\overline{A}, \overline{B}\right)\right)\right)}{\lambda d_H\left(\underline{A}, \underline{B}\right) + (1-\lambda)\left(d_H\left(\overline{A}, \overline{B}\right)\right)}$$

with $\lambda \in [0, 1]$ and where $d_H(A, B) = \int_{\mathbb{R}} |\mu_A(x) - \mu_B(x)| dx$ is the Hamming distance between two fuzzy numbers. The value of the relation is defined as $P_{N^\lambda}(A, B) = 0.5$, when $\lambda d_H\left(\underline{A}, \underline{B}\right) + (1-\lambda)\left(d_H\left(\overline{A}, \overline{B}\right)\right) = 0$.

3. Main results

Brunelli and Mezei estimated the degree of similarity between ranking methods using the Spearman index. Consequently, they proposed that methods Y_2 and $N^{0.5}$ induce the same ranking for all the trapezoidal fuzzy numbers. We prove that the conjecture is true for all fuzzy numbers in this section.

In order to show that methods Y_2 and $N^{0.5}$ induce the same ranking for all fuzzy numbers, the next lemmas and proposition are provided.

Lemma 1. *(See [11].) Let A and B be fuzzy numbers. Then we have for* $x \in \mathbb{R}$*:*

(1) $\mu_{\widetilde{\max}(\underline{A}, \underline{B})}(x) = \min\left(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)\right)$;
(2) $\mu_{\widetilde{\min}(\underline{A}, \underline{B})}(x) = \max\left(\mu_{\underline{A}}(x), \mu_{\underline{B}}(x)\right)$;
(3) $\mu_{\widetilde{\min}(\overline{A}, \overline{B})}(x) = \min\left(\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)\right)$;
(4) $\mu_{\widetilde{\max}(\overline{A}, \overline{B})}(x) = \max\left(\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)\right)$;

Lemma 2. *(See [4].) Let A be a nonnegative fuzzy number. Then*

$$\int_0^1 a_\alpha^- d\alpha = a_1^- - \int_{a_0^-}^{a_1^-} \mu_{\underline{A}}(x)dx \quad \text{and} \quad \int_0^1 a_\alpha^+ d\alpha = a_1^+ + \int_{a_1^+}^{a_0^+} \mu_{\overline{A}}(x)dx.$$

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