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Existence of interval, triangular, and trapezoidal approximations of fuzzy numbers under a general condition

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Abstract

Recently, many scholars studied approximations of fuzzy numbers by specific fuzzy numbers under preservation of some operators. In fact, these approximations may not exist for some linear operators. The purpose of this paper is to study necessary and sufficient conditions of linear operators which are preserved by interval, triangular, symmetric triangular, trapezoidal, or symmetric trapezoidal approximations of fuzzy numbers. In addition, an effective method for solving such problems is proposed.

Keywords: Fuzzy numbers; Trapezoidal approximations; Triangular approximations; Linear operators; Expected interval; Value; Ambiguity

1. Introduction

In practice, it is used to approximate general fuzzy numbers by specific fuzzy numbers so as to simplify calculation. Recently, the nearest interval, triangular, symmetric triangular, trapezoidal, or symmetric trapezoidal fuzzy number to a fuzzy number without any constraints had been studied, which are called the interval, triangular, symmetric triangular, trapezoidal, or symmetric trapezoidal approximations of fuzzy numbers [1–3,11,12,15,16,19,20,22,27,29,33,34,38]. Some more general approximations are studied, too [6,7,18,26,30,35–37]. Sometimes, it is necessary that these approximations should preserve some attributes. Therefore, many scholars investigated approximations of fuzzy numbers under preservation of some (linear) operators, which are called *approximations preserving* (*linear*) *operators* in the present paper. The following approximations preserving operators had been studied: trapezoidal approximation preserving the expected interval proposed by Grzegorzewski and Mrówka [24,25] and improved in many papers [5,21,23,32,34], trapezoidal approximation preserving cores proposed by Grzegorzewski and Stefanini [26] and further studied by Abbasbandy and Hajjari [4], trapezoidal approximation preserving the value and ambiguity proposed by Ban et al. [8], trapezoidal approximation preserving ambiguity proposed by Ban and Coroianu [13], symmetric trapezoidal approximation preserving the x-centroid proposed by Wang and Li [31], and triangular approximation preserving the x-centroid proposed by Li et al. [28]. Some properties of this type of approximations are also studied

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[9,10,17]. In [14], the authors proved that not all (linear) operators can be preserved by trapezoidal approximations of fuzzy numbers. They showed a necessary and sufficient condition of linear operators for such approximation (see the following Theorem 4.1 in Section 4). In that paper, they would like to know what the necessary and sufficient conditions of linear operators for other approximations are, such as interval, triangular, symmetric triangular, and symmetric trapezoidal approximations. In the present paper, we fully solve these problems. In fact, the proposed method can be applied to any other approximations.

The present paper is organized as follows. In Section 2, we introduce linear operators of fuzzy numbers. In Section 3, we show the necessary and sufficient conditions of linear operators for any approximation. In Section 4, we apply the proposed theorem in Section 3 to examine interval, triangular, symmetric triangular, and symmetric trapezoidal approximations, and a conclusion is given in Section 5.

2. Linear operators on fuzzy numbers

A fuzzy number $\tilde{A}$ with α-cuts $\tilde{A}^\alpha = [A_L(\alpha), A_U(\alpha)]$, $\alpha \in [0,1]$, is a fuzzy set on the real line, denoted by $\mathbb{R}$, which fulfills that:

(1) $A_L = A_L(\alpha)$ and $A_U = A_U(\alpha)$ are both left continuous functions from [0,1] to $\mathbb{R}$,
(2) $A_L = A_L(\alpha)$ is increasing and $A_U = A_U(\alpha)$ is decreasing, and
(3) $A_L(1) \le A_U(1)$.

Let $F(\mathbb{R})$ denote the set of all fuzzy numbers. Recall that a fuzzy number $\tilde{A}$ with α-cuts $\tilde{A}^\alpha = [A_L(\alpha), A_U(\alpha)]$, $\alpha \in [0,1]$, is called *trapezoidal* (*interval*) if its degrees of $A_L(\alpha)$ and $A_L(\alpha)$ are both less than or equal to 1 (resp., 0). If it additionally satisfies $A_L(\alpha)(1) = A_U(\alpha)(1)$ it is called *triangular*. And, $\tilde{A}$ is *symmetric* if $A_L(\alpha) + A_U(\alpha)$ is constant. Let $F_T(\mathbb{R})$ denote the set of all trapezoidal fuzzy numbers, $F_{T^s}(\mathbb{R})$ the set of all symmetric trapezoidal fuzzy numbers, $F_\Delta(\mathbb{R})$ the set of all triangular fuzzy numbers, $F_{\Delta^s}(\mathbb{R})$ the set of all symmetric triangular fuzzy numbers, and $F_I(\mathbb{R})$ the set of all interval fuzzy numbers. Recall that, if a subset C of any vector space V satisfies

$$u + v \in C \quad \text{and} \quad ru \in C,$$

for all $u, v \in C$ and all $r \in \mathbb{R}^+ \cup \{0\}$, then it is called a *cone*. In [35], the author showed that the set of all fuzzy numbers can be embedded into a vector space (in fact, it is a complete inner product space).

fact 2.1. *All of $F(\mathbb{R})$, $F_T(\mathbb{R})$, $F_{T^s}(\mathbb{R})$, $F_\Delta(\mathbb{R})$, $F_{\Delta^s}(\mathbb{R})$, and $F_I(\mathbb{R})$ are cones.*

A function f from a vector space V to $\mathbb{R}$ is called *linear* if it fulfills

$$f(u+v) = f(u) + f(v) \quad \text{and} \quad f(rv) = rf(v),$$

for all $u, v \in V$, $r \in \mathbb{R}$.

Proposition 2.2. *Let V be a vector space, $f : V \to \mathbb{R}$ be any linear operator, and C be any cone in V. Then, $f(C)$ equals one of the following subsets: $\mathbb{R}$, $(-\infty, 0]$, $[0, \infty)$, $\{0\}$.*

Proof. Suppose that f is nonzero on C, i.e. there exists a $v \in C$ such that $f(v) \neq 0$. Suppose that $f(v) > 0$. Let $r \in \mathbb{R}^+ \cup \{0\}$. Since C is a cone, we have $\frac{r}{f(v)} \cdot v \in C$, which implies

$$f(\frac{r}{f(v)} \cdot v) = \frac{r}{f(v)} \cdot f(v) = r.$$

Hence, $f(C)$ contains $[0, \infty)$. In the same way, if $f(v) < 0$ then $f(C)$ contains $(-\infty, 0]$. This completes the proof. □

Corollary 2.3. *Let f be any linear operator on $F(\mathbb{R})$. Then, each of $f(F(\mathbb{R}))$, $f(F_T(\mathbb{R}))$, $f(F_{T^s}(\mathbb{R}))$, $f(F_\Delta(\mathbb{R}))$, $f(F_{\Delta^s}(\mathbb{R}))$, and $f(F_I(\mathbb{R}))$ is equal to one of the following subsets: $\mathbb{R}$, $(-\infty, 0]$, $[0, \infty)$, $\{0\}$.*

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