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Computation of the output of a function with fuzzy inputs based on a low-rank tensor approximation [☆]

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Abstract

We apply a derivative-free optimization method based on novel low-rank tensor methods to the problem of propagating fuzzy uncertainty through a continuous real-valued function. Adhering to Zadeh's extension principle, such a problem can be reformulated as a sequence of optimization problems over nested search spaces. The optimization method we use is based on a low-rank tensor approximation of the function sampled on a grid and a search for the minimal and maximal entries in this low-rank tensor. In contrast to classical fuzzy uncertainty propagation methods, such as the vertex method and the transformation method, the method we propose does not exhibit an inherent exponential scaling for increasing dimension of the search space. Obviously, no derivative-free optimization algorithm can exist which shows sub-exponential scaling with the dimension for all possible continuous functions. The algorithm that we present here, however, can exploit a specific type of structure and regularity (beyond continuity) that is often present in real-world optimization problems. We illustrate this with some high-dimensional numerical examples where the presented method clearly outperforms some established derivative-free optimization codes.

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Keywords: Fuzzy numbers; Zadeh's extension principle; α-cut approach; Global optimization; Grid search; Low-rank tensors

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1. Introduction

This paper deals with the problem of propagating fuzzy uncertainty through a continuous real-valued function, i.e., applying such a function to a fuzzy number or a vector of fuzzy numbers. Such numbers and vectors are examples of fuzzy sets, a concept introduced by L. Zadeh in [1]. Intuitively, fuzzy sets are sets of elements, each with an associated membership level which is a value in the range of zero to one. For example, some elements may definitely belong to the fuzzy set, in which case their membership level equals one. Other elements may be definitely outside of the fuzzy

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set and have level zero, whereas still others can be in the fuzzy set to a certain degree or to a certain extent; the latter have a level between zero and one.

Fuzzy numbers are fuzzy sets that are defined over $\mathbb{R}$. They have found extensive application in the domain of uncertainty quantification. There, they are used to model epistemic uncertainties, i.e., uncertainties which are due to vagueness or lack of information. For example, a value which is "about three", or which is "somewhere in between four and five" can be represented as a fuzzy set. In [2] an arithmetic with fuzzy numbers was defined, and in [3] the concept of a function of a fuzzy number was introduced.

Intuitively, a real-valued function of a fuzzy number is again a fuzzy number. It is the set of all possible values that can be found by applying the function to each of the elements in the support of the original fuzzy number. Each of the resulting values receives a membership level, the value of which is determined on the basis of a central axiom of fuzzy set theory, the Zadeh extension principle. This principle allows one to define a calculus for fuzzy numbers, and is at the basis of, for example, the definition of the concept of a solution to a fuzzy differential equation using sample path-based fuzzy fields [4–9]. Such differential equations are currently under intense investigation, e.g., in the engineering literature [10–12], where they are found to be very well suited to assess the effect of vagueness on the model parameters in early engineering design phases. Zadeh's extension principle, together with some essential background information on fuzzy sets and numbers, will be recalled in §2.

It is well known that the problem of applying a continuous real-valued function to a fuzzy number or a vector of fuzzy numbers can be reformulated as a sequence of optimization problems over nested search spaces which are contained within the support of the fuzzy input. This is the so-called α-cut approach [13]. In the special case that the entries in the input vector are fuzzy numbers which are non-interactive, i.e., independent, the search spaces reduce to hyperrectangles.

Global optimization is generally considered to be a difficult problem. The optimization of a quadratic polynomial over a hyperrectangle, for example, is known to be a NP-hard problem [14]. In the derivative-free optimization setting, information-based complexity results show that the optimization of a Lipschitz continuous function over a hyperrectangle is intractable [15]. The number of needed function evaluations to reach a certain accuracy for all possible Lipschitz continuous functions scales exponentially with the dimension. In fact, no algorithm can do better than grid search, i.e., sampling the function on a regular grid and selecting the extremal function value.

Real-world optimization problems, however, tend to exhibit much more structure. This explains the success of the many existing alternative global optimization algorithms. The key issue, here, is to exploit the structure in the problem. We apply an optimization method as described in [16]. In a first step, a low-rank tensor approximation of a grid sampling of the function is constructed. This is followed by a search for the extremal entry in this low-rank tensor, which then serves as an estimate of the optimum. Because the function has to be optimized over a sequence of nested hyperrectangles, we will choose to construct the low-rank tensor approximation once and for all. Parts of this tensor are then used to estimate the optima over the different hyperrectangles.

The algorithm we use to construct the low-rank tensor approximation is the one from [17]. For an overview of other such algorithms which construct a low-rank tensor approximation from a selection of grid samples of a function, we refer to [18]. The search for the extremal entry in a low-rank tensor will be done with the algorithm as implemented in the MATLAB $\mathcal{H}$-Tucker tensor toolbox [19]. Other such algorithms can be found in the MATLAB TT-toolbox from Oseledets and in [16].

The structure of the paper is as follows. Section 2 recalls some necessary background about fuzzy sets, fuzzy numbers, and fuzzy arithmetic and overviews the current practice of numerical methods for the propagation of fuzzy uncertainty through continuous real-valued functions. Section 3 introduces our method based on low-rank tensors, together with an error estimate and a computational complexity estimate. In Section 4, we demonstrate the effectiveness of the technique on some challenging problems. Finally, some concluding remarks end the paper in Section 5.

2. Some preliminaries on fuzzy sets, numbers and calculus

We start with describing the main concepts of fuzzy set theory and the corresponding terminology and notation that we will use throughout the paper. For a more elaborate introduction to fuzzy sets and arithmetic we refer to the following two books [20,21].

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