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Fuzzy rule-based models with interactive rules and their granular generalization

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Abstract

Processing realized in fuzzy rule-based models is associated with matching individual rules with the existing input data and aggregation of the levels of matching and conclusions of the rules. Rules are processed as individual entities. In this study, we introduce an augmentation of fuzzy models by facilitating interaction among the rules leading to more flexible type membership functions of fuzzy sets forming conditions of the rules (thus resulting in substantially advanced topology of the partition of the input space). Different ways of realizing interaction among the rules are studied. In the sequel, we develop a granular fuzzy model implied by the rule-based model showing how granular parameters of the original rule-based model enhance its quality expressed in terms of coverage of experimental data. The two evaluation criteria of the constructed granular model, namely coverage and specificity are studied. Experimental results are reported for a series of publicly available data.

Keywords: Fuzzy rule-based models; Interaction among rules; Granular model; Coverage; Specificity; Deep learning; Allocation of information granularity

1. Introductory notes

Rule-based models, and fuzzy rule-based models in particular, are granular constructs with information granules forming their condition and conclusion parts (Mamdani architecture of the model) or conditions parts only (Takagi–Sugeno topology). A number of approaches have been proposed to generate fuzzy rule-based models [1–4]. There have been a spectrum of studies, which are focused on analysis of fuzzy models and elaborate on knowledge representation realized by such modules. There are also discussions reported on the quality of the rules and investigations

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of various reasoning or aggregation mechanisms [5–8]. In the literature, possible interactive relationships between the rules are usually discussed and realized by means of mechanisms of aggregation or decomposition, and subsequently implemented to improve the performance of fuzzy models, fuzzy classifiers and decision making schemes [9–12,17,18]. One can refer to studies devoted to hierarchical architectures of fuzzy rule-based models where the models are structured on a basis of a hierarchy of rules with the rules exhibiting different levels of specificity (depending upon their location within the hierarchy of the rules), see [13,14,25]. In particular, the concept of formation and interaction within hierarchies of fuzzy models is related to some aspects of deep learning, in which we inherently encounter a collection of layers of processing facilitating a formation of additional features emerging at the higher levels of abstraction [15,16]. Likewise, the available design procedures are also highly diversified both in terms of general development strategies as well as the detailed optimization vehicles being used in their construction [17–19].

Interestingly, within the plethora of architectural enhancements of fuzzy rule-based models, a question of interaction among the rules and a possible usage of the interaction mechanism towards the enhancements of the performance of the rule-based models has not been studied. By interaction among the rules we mean a mechanism of engaging existing fuzzy sets standing the corresponding rules in a formation of new fuzzy sets so that the fuzzy sets formed in this manner give rise to more efficient structure of the overall model. This is an original and promising development direction whose main features and associated improvements are worth considering. Our ultimate objective in this study is to introduce the concept of interactive rules, study their properties and show ways they deliver additional functionality to fuzzy models.

We start (in Section 2) with a brief recollection of some design aspects of fuzzy rule-based models; here we highlight some structural aspects of the model as well. Several ways of forming interaction among the rules are discussed in Section 3; here we also elaborate on expansion and contraction effects related with the number of fuzzy sets being formed as a result of the introduced transformations. We also discuss ways of optimizing the remaining parameters of the augmented rule-based model. Granular generalization of the (numeric) rule-based model is introduced in Section 4. In Section 5, we present results of comprehensive experiments while conclusions are covered in Section 6.

2. Fuzzy rule-based model: structure and design

In what follows, we briefly recall the main ideas behind rule-based models along with the main design strategies.

2.1. Topology of the rules and the structure of the model

A generic architecture of the model dwells upon a collection of c rules assuming the following form

$$\text{if } \boldsymbol{x} \text{ is } \boldsymbol{A}_i \text{ then } y_i \tag{1}$$

$i = 1, 2, \ldots, c$ where $\boldsymbol{A}_i$ are information granules (fuzzy sets) defined in the n-dimensional input space and y_i are some numeric representatives distributed across the output space. For any $\boldsymbol{x}$ in the input space, the inference (reasoning) scheme is commonly realized by determining the corresponding activation levels of the rules and summing up the contributions of the conclusion parts as shown below

$$y = \sum_{i=1}^{c} \boldsymbol{A}_i(\boldsymbol{x})\bar{y}_i \tag{2}$$

The condition parts of the rules are formed by fuzzy sets. More specifically, $\boldsymbol{A}_i$ standing in (2) denotes a fuzzy set expressed in the n-dimensional input space, that is $\boldsymbol{A}_i : \boldsymbol{R}^n \to [0, 1]$, and $\bar{y}_i$ are constants (constant functions) denoting the conclusion values of the corresponding rules, namely $\bar{y}_i : \boldsymbol{R} \to \boldsymbol{R}$. It is worth noting that the rules of this format are a realization of the Mamdani type of rules [1] coming in the form "if $\boldsymbol{x}$ is $\boldsymbol{A}_i$ then y is $\boldsymbol{D}_i$" with the decoding being implemented by taking the numeric representative of the fuzzy set of conclusion D_i (such as its mean or modal value). These rules can be regarded as the simplest version of the Takagi–Sugeno rules [2,24] "if $\boldsymbol{x}$ is $\boldsymbol{A}_i$ then y is $f_i(\boldsymbol{x})$" with the functional format of the conclusion viz. the local functions being the constant ones ($f_i(\boldsymbol{x}) = \bar{y}_i$).