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Some properties of solutions for a class of semi-linear uncertain dynamical systems ☆

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Received 6 March 2014; received in revised form 22 May 2016; accepted 23 May 2016

Abstract

In this paper some properties of solutions are obtained for semi-linear fuzzy differential equations understood as differential inclusions, i.e., semi-linear uncertain dynamical systems. The definitions of the solution to the semi-linear uncertain dynamical system and its corresponding big solution are given. The existence of the two kinds of solutions is established, and the inclusion relation between them is also obtained.
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Keywords: Fuzzy number; Semi-linear uncertain dynamical system; Green's function; Differential inclusion

1. Introduction

Fuzzy differential equations are used to solve practical problems with uncertainty, which come from physics, control theory and neural networks, etc. Especially, periodic problems, anti-periodic problems and periodic doubling problems are widely used in the real world. For example, the following fuzzy differential equation is always taken into consideration:

$$x' + \lambda x = f(t, x),\ \ x(0) = ax(T),$$

where $f : [0, T] \times \mathbf{E}^1 \to \mathbf{E}^1$, and $\lambda,\ a \in \mathbf{R}$.

At present, research on solutions to fuzzy differential equations follows three methodologies: H-derivatives and Bede's generalized derivatives which are generalized from H-derivatives (see [2,3,8,12,17,18,21,24,25]); Zadeh's Extension Principle (see [6,7,26,27]); differential inclusions (see [1,9–11,13,19,20,23]). And the three different methodologies have different background theories (see [5,9,11]). The H-derivatives method is one popular way to deal with fuzzy differential equations. But for the above problem, no periodic solutions can be found by using H-derivatives when $a = 1$, $\lambda > 0$ (see the results in [4,9–12,23]). In the sense of H-derivatives, this problem also has no solutions

☆ This work was supported by the National Natural Science Foundation of China (Grant Nos. 11271099 and 11471088).
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http://dx.doi.org/10.1016/j.fss.2016.05.013
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Please cite this article in press as: R. Dai, M. Chen, Some properties of solutions for a class of semi-linear uncertain dynamical systems, Fuzzy Sets Syst. (2016), http://dx.doi.org/10.1016/j.fss.2016.05.013

if $a = -1$ or $0 \leq a \leq 1$. The idea in [10,13,19], which considers fuzzy differential equations as uncertain dynamical systems, can handle the above periodic problem better. So in this case, the method of differential inclusions can solve these problems at the practical level more effectively.

In Ref. [10], by using differential inclusions, the semi-linear fuzzy differential equation:

$$x' + \lambda x = f(t, x),\ x(0) = x(T)\ \ (\text{where } f : [0, T] \times \mathbf{E}_c \to \mathbf{E}_c,\ \mathbf{E}_c \subset \mathbf{E}^1, \lambda > 0)$$

has been considered as the periodic behavior of the semi-linear uncertain dynamical system:

$$\xi' + \lambda\xi \in f(t, \xi),\ \ \xi(0) = \xi(T),$$

where $f : [0, T] \times \mathbf{R} \to \mathbf{E}_c$ and for $\eta \in \mathbf{R}$, $u \in \mathbf{E}_c$, $\eta \in u$ means that the membership function of u satisfies $\mu_u(\eta) > 0$. The periodic solutions of this semi-linear uncertain dynamical system have been obtained.

Similarly to [10], we consider the following problem of solving the semi-linear fuzzy differential equation:

$$x' + \lambda x = f(t, x),\ x(0) = ax(T)\ (\text{where } f : [0, T] \times \mathbf{E}_c \to \mathbf{E}_c,\ \lambda > 0,\ a > 0)$$

as the following problem of the semi-linear uncertain dynamical system:

$$\xi' + \lambda\xi \in f(t, \xi),\ \ \xi(0) = a\xi(T)$$

where $t \in [0, T]$ $(T > 0)$, $\lambda > 0$, $f : [0, T] \times \mathbf{R} \to \mathbf{E}_c$, $a > 0$, $a \neq e^{\lambda T}$. When $a = 1$, it's the same problem as [10]'s. But in this paper, conditions for proving the existence and other properties of solutions are more simple and clear. And the case of $a > 0$, $a \neq e^{\lambda T}$ has also been solved.

The rest of this paper is organized as follows. In Section 2, basic theories are given. In Section 3, the solution and the big solution to the problem of the semi-linear uncertain dynamical systems are defined. The big solution is the solution to the corresponding integral equation that has the extended forcing function. By using the differential inclusions and the Green function, the existence of the solution and the big solution is established under some conditions, and the inclusion relation between the solution and its corresponding big solution is also obtained. In Section 4, the conclusion and future directions are put forward.

2. Preliminaries

The basic background theories used in this paper are presented as follows.

Let $\mathbf{D}^1$ be the set of upper semicontinuous normal fuzzy sets with compact supports in $\mathbf{R}$ and $\mathbf{E}^1$ be the set of fuzzy convex subsets of $\mathbf{D}^1$ (see [13]).

Stacking Theorem *([13]). Let* $\{A_\alpha \subset \mathbf{R} |\ 0 \leq \alpha \leq 1\}$ *be a class of nonempty compact sets satisfying*

(i) $A_\beta \subset A_\alpha$ $(0 \leq \alpha \leq \beta \leq 1)$,

(ii) $A_\alpha = \bigcap_{n=1}^{\infty} A_{\alpha_n}$ *for any nondecreasing sequence* $\{\alpha_n\}$ *in* $[0, 1]$ *satisfying* $\alpha_n \to \alpha$.

Then there exists $v \in \mathbf{D}^1$ *such that* $[v]^\alpha = A_\alpha$ $(0 \leq \alpha \leq 1)$. *Especially if* A_α *is convex,* $v \in \mathbf{E}^1$. *On the other hand, if* $v \in \mathbf{D}^1$, *the level set* $[v]^\alpha$ *satisfies (i) and (ii) above. If* $v \in \mathbf{E}^1$, $[v]^\alpha$ *is convex.*

Definition 2.1 *([9]).* Let $\mathbf{E}_c = \{u \in \mathbf{E}^1 |\ u_1(\alpha) = \min[u]^\alpha,\ u_2(\alpha) = \max[u]^\alpha$ are continuous on $[0, 1]\}$, i.e. $u \in \mathbf{E}_c$ satisfies the following conditions (i)–(v):

(i) u is normal, i.e., $\exists m \in \mathbf{R}$ such that $u(m) = 1$,
(ii) $[u]^0 = cl\{\xi \in \mathbf{R} | u(\xi) > 0\}$ is bounded in $\mathbf{R}$,
(iii) u is fuzzy convex in $\mathbf{R}$,
(iv) u is upper semicontinuous on $\mathbf{R}$,
(v) Denote $[u]^\alpha = \{\xi \in \mathbf{R} | u(\xi) \geq \alpha\}$ $(0 < \alpha \leq 1)$, $u_1(\alpha) = \min[u]^\alpha$, $u_2(\alpha) = \max[u]^\alpha$ $(\alpha \in [0, 1])$, then $u_1(\alpha), u_2(\alpha)$ are continuous on $[0, 1]$.

We call $u \in \mathbf{E}_c$ continuous fuzzy number and fuzzy number in abbreviation.

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