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A new Bernoulli wavelet method for accurate solutions of nonlinear fuzzy Hammerstein–Volterra delay integral equations

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Abstract

In this article, Bernoulli wavelet method has been developed to solve nonlinear fuzzy Hammerstein–Volterra integral equations with constant delay. This type of integral equation has a particular case the fuzzy variant of a mathematical model from epidemiology. Bernoulli wavelets have been generated by dilation and translation of Bernoulli polynomials. The properties of Bernoulli wavelets and Bernoulli polynomials are first presented. The present wavelet method reduces these integral equations to a system of nonlinear algebraic equations and again these algebraic systems have been solved numerically by Newton's method. Convergence analysis of the present method has been discussed in this article. Also the results obtained by present wavelet method have been compared with that of by B-spline wavelet method. Some illustrative examples have been demonstrated to show the applicability and accuracy of the present method.

Keywords: Hammerstein–Volterra integral equation; Fuzzy delay integral equation; Bernoulli polynomials; Bernoulli wavelets

1. Introduction

The study of fuzzy integral equations and fuzzy differential equations is an emerging area of research for many authors. Originally, the concept of fuzzy sets was first introduced by Zadeh [1,2]. The development of fuzzy integral equations was first invented by Kaleva [3] and Seikkala [4]. In recent years, many researchers have focused their interest on this field and published many articles which are available in literature. Many analytical methods like Adomian decomposition method [5], homotopy analysis method [6], and homotopy perturbation method [7] have been used to solve fuzzy integral equations. There are available many numerical techniques to solve fuzzy integral equations. The method of successive approximations [8,9], quadrature rule [10], Nystrom method [11], Lagrange interpolation [12], Bernstein polynomials [13], Chebyshev interpolation [14], Legendre wavelet method [15], sinc function [16], residual minimization method [17], fuzzy transforms method [18], and Galerkin method [19] have been applied to solve fuzzy integral equations numerically. Recently, Sadatrasoul et al. [20] have solved nonlinear

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fuzzy integral equations by applying iterative method. Many theories related to fuzzy fractional functional integral and differential equations have been included in [21] and convergence in measure theorem for nonlinear integral functionals has been provided in [22]. Existence of solutions to fuzzy differential equations with generalized Hukuhara derivative via contractive-like mapping principles has been presented by Villamizar-Roa et al. [23]. A classical solution of fuzzy boundary value problem has been given in [24]. The Cauchy problem for complex fuzzy differential equations has been solved in [25]. Hybrid block-pulse functions and Taylor series method [26] have been applied to solve nonlinear fuzzy Fredholm integral equations of the second kind and also linear Fredholm fuzzy integral equations of the second kind have been solved by artificial neural networks [27].

In this paper, the nonlinear Hammerstein–Volterra fuzzy integral equation with constant delay has been considered and it has the following form

$$x(t) = \begin{cases} g(t) + \int_{t-\tau}^{t} H(t,s)F(s,x(s))ds, & t \in [0,T] \\ \Phi(t), & t \in [-\tau, 0], \end{cases} \tag{1}$$

where $\tau > 0$, $T > 0$ and g, F, and Φ are known fuzzy valued functions. This type of delay integral equation occurs in the study of the epidemic model [28] where $H(t,s) = P(t-s)$. The boundedness of the positive solutions of this model is studied in [28], while the existence of the positive solutions in the case when $g(t) = 0$ and $H(t,s) = 1$ is studied in [29,30]. This model has been solved by successive approximation method by Bica et al. [31]. In this present study, the nonlinear Hammerstein–Volterra fuzzy delay integral equation has been solved by Bernoulli wavelet method (BWM) and the obtained results then compared with the results obtained by B-spline wavelet method (BSWM). The present method reduces the integral equation into system of algebraic equations and then the algebraic system can be solved numerically by any usual method. Previously, Bernoulli wavelets have been applied to solve fractional order integral and differential equations by Razzaghi et al. [32]. Linear semi-orthogonal B-spline wavelets have been applied to solve many linear and nonlinear integral equations [33–35]. Many other wavelet methods [36,37] are also available in literature for solving fuzzy integral equations. The numerical solutions of delay integral equations have been studied by many researchers in [38,39]. Bellour et al. [38] have proposed a Taylor collocation method for solving delay integral equations and Dehghan et al. [40] have solved biological model problem involving constant delay integro-differential equations.

The rest of the paper has been organized as follows: in section 2, we present some preliminaries and notations useful for fuzzy integral equations. In section 3, we discuss the properties of Bernoulli wavelets and function approximation. In section 4, we establish the method for solving nonlinear Hammerstein–Volterra fuzzy delay integral equation. In section 5, we present the convergence analysis of Bernoulli wavelet method. Section 6 deals with the illustrative examples which show the efficiency and accuracy of the presented method.

2. Preliminaries of fuzzy integral equation

In this section the most basic notations used in fuzzy calculus are introduced. We start with defining a fuzzy number.

Definition 2.1. (See Ref. [41].) A fuzzy number u is represented by an ordered pair of functions $\left(\underline{u}(r), \overline{u}(r)\right)$; $0 \le r \le 1$ which satisfying the following properties.

1. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function.
2. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function.
3. $\underline{u}(r) \le \overline{u}(r), \quad 0 \le r \le 1$.

For arbitrary $u(r) = \left(\underline{u}(r), \overline{u}(r)\right)$, $v(r) = \left(\underline{v}(r), \overline{v}(r)\right)$ and $k > 0$, we define addition $(u + v)$ and scalar multiplication by k as

i. $\left(\underline{u+v}\right)(r) = \underline{u}(r) + \underline{v}(r)$
ii. $\left(\overline{u+v}\right)(r) = \overline{u}(r) + \overline{v}(r)$
iii. $\left(\underline{ku}\right)(r) = k\underline{u}(r)$, $\left(\overline{ku}\right)(r) = k\overline{u}(r)$