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# Constraint heterogeneous concept lattices and concept lattices with heterogeneous hedges

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**Abstract**

The paper deals with the isomorphism between the constraint heterogeneous concept lattices and concept lattices with heterogeneous hedges. The essential point of the former approach encompasses the full diversification of data structures within a formal context. In particular, we use a different complete lattice for diverse objects, a different complete lattice for diverse attributes and a different poset for diverse matrix fields. The latter framework with heterogeneous hedges results in a reduction in the size of the corresponding concept lattice. We present the properties of constraint heterogeneous approach that is associated with the fixpoints of hedges and we add remarks to the related studies.


*Keywords:* Formal concept analysis; Algebra; Heterogeneous concepts; Hedges

## 1. Introduction

Formal concept analysis [23] scrutinizes an object-attribute block of the binary relational data under the notion of a formal concept. The classical approach mines the formal concepts that are generated from the binary relation and several fuzzifications have been proposed [6,13,41,42]. A fuzzy setting considers a degree to which an object has a particular attribute and in particular, the extensions of $L$-fuzzy concept lattices encompass generalized concept lattices [28], multi-adjoint concept lattices [32,35,21], Galois connectional concept lattices [39,40], interval-valued $L$-fuzzy contexts [1]. The extensions taking into account the structures of idempotent semifields and their completions [43] or multiply diversification of structures [3] are another answer to generalizations in formal concept analysis. Concerning a categorical approach, [30,31] present the intercontextual relationships of formal contexts. Formal concept analysis, in general, is an interesting research area that provides theoretical foundations, fruitful methods, algorithms and underlying applications in many areas and has been investigated in relation to various disciplines [18].

Krajči [29] proved that generalized concept lattices cover concept lattices with hedges [9,10]. Generalized concept lattices are based on three sets of truth degrees as a kind of a three-sorted residuated structure with a specific isotone

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aggregation function [7]. Heterogeneous hedges within a context are introduced by Bělohlávek and Vychodil [11] as the extension for $L$-fuzzy formal context with the same hedge for the set of objects and the same hedge for the set of attributes [9,10]. The reduction in size of the concept lattice is demonstrated and the theorem which shows that stronger hedges lead to a smaller concept lattice is proved. On the other hand, isotone fuzzy Galois connections related to $L$-fuzzy formal context with hedges are studied by Konečný [25].

Non-commutative conjunctors from fuzzy logic programming [36–38] are the cornerstone for the framework of multi-adjoint concept lattice [35]. In [33], Medina and Ojeda-Aciego introduce the notion of $L$-connected lattices. $L$-connection of two complete lattices generalizes the notion of hedges in formal concept analysis. In [26], Konečný et al. describe that the selection of complete sup-semilattice preserves extents of the original multi-adjoint $L$-connected concept lattice and it is shown how their results generalizes the propositions about subcontexts from [23]. The construction of adjoint triples using hedges and the conditions for their generation are formulated in [26].

In our paper, we would like to clarify the relationship between the heterogeneous concept lattices [3] and the novel extension of concept lattices with heterogeneous hedges given by Bělohlávek and Vychodil [11]. Our aim is to continue our research on heterogeneous data in formal concept analysis and to fill in the gaps of our previous studies. First, we investigate the properties of the constraint heterogeneous concept lattices with respect to hedges. A translation process between fuzzy setting and ordinary crisp setting provides a way of stating relationship between heterogeneous concept lattices and concept lattices with heterogeneous hedges.

We state that Butka et al. in [15] describe a representation of fuzzy concept lattices (represented as crisp complete lattices) in the framework of classical concept lattices, as well. They involve the principal ideal as the structure for a translation process from fuzzy formal context into a binary formal context. On the contrary, we work with an ordinary subset of the Cartesian product defined directly from the particular fuzzy membership function. Moreover, we create the heterogeneous concept lattice first and then find a representation of this heterogeneous concept lattice as a classical concept lattice, therefore the principal ideal can be omitted in our paper.

In particular, Section 2 provides preliminaries on concept lattices with heterogeneous hedges. Section 3 describes the extended observations about the interpretation of hedges in formal concept analysis and investigates the properties of fuzzy membership functions with respect to translation process. Section 4 recalls the basic notions of our heterogeneous approach; Section 5 explains how to construct the isomorphism on constraint heterogeneous concept lattices. Remarks on related studies are included in Section 6. We argue about the university admission tests represented by the constraint heterogeneous concept lattice in a working example in Section 7. Finally, conclusions are summarized.

## 2. Concept lattices with heterogeneous hedges

Formal concept analysis with heterogeneous hedges is intensively studied in [11]. This new approach has been introduced by Bělohlávek and Vychodil. A new environment extends the one from [9,10]. We remind the basic notions.

Let $L$ be a set of truth degrees, then $\langle L, \vee, \wedge, \otimes, \rightarrow, 0, 1\rangle$ forms a complete residuated lattice in which $\langle L, \vee, \wedge, 0, 1\rangle$ is a complete lattice with 0 and 1 being the least and greatest element of $L$, $\langle L, \otimes, 1\rangle$ is a commutative monoid, $\otimes$ and $\rightarrow$ satisfy adjointness. Consider a set of objects $X$, a set of attributes $Y$ and a binary fuzzy relation $R$ between $X$ and $Y$ such that $R(x, y) \in L$. We introduce a definition of a hedge from [10], which is close to a logical connective from [24].

**Definition 1.** (See [10].) A hedge is a unary mapping $*$ on $L$ such that for each $a, b \in L$:

$$1^* = 1,$$

$$a^* \leq a,$$

$$(a \rightarrow b)^* \leq a^* \rightarrow b^*,$$

$$a^{**} = a^*.$$

Denote the set of all fixpoints of $*$ in $L$ by $\text{fix}(*) = \{a \in L : a^* = a\}$.

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