ARTICLE IN PRESS FSS:6941

Available online at www.sciencedirect.com

ScienceDirect

Fuzzy Sets and Systems ••• (••••) •••–•••

FUZZY sets and systems

www.elsevier.com/locate/fss

Fuzzy inner product spaces

M. Saheli *, S. Khajepour Gelousalar

Department of Mathematics, Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran

Received 23 February 2014; received in revised form 7 November 2015; accepted 12 November 2015

Abstract

In this paper, we show that all results obtained recently on fuzzy inner product spaces are obvious. To achieve this goal, we modify the definition of the fuzzy inner product space and then we mention and prove some monotonicity and orthogonality properties of the new fuzzy inner product space. Finally, we describe fuzzy inner products in terms of families of crisp inner products. Namely, any fuzzy inner product corresponds to a family of crisp inner products and vice versa.

Keywords: Fuzzy norm; Fuzzy inner product; Fuzzy Hilbert space

1. Introduction

A definition of fuzzy inner product space and its associated fuzzy norm function have been firstly given by Biswas [3], El-Abyad and Hamouly [4]. Later, Kohli and Kumar [8] modified the definition of inner product space introduced in [3]. Recently, different types of fuzzy inner product space have been presented whose corresponding fuzzy norm is the norm introduced by Bag and Samanta [1], for more details see [6,9,10]. Hasankhani et al. [7], introduced a notion of a fuzzy inner product space on a linear space so that its corresponding fuzzy norm is of the Felbin type [5]. A notion of fuzzy n-inner product space has been used by Vijayabalaji [12,13].

In this paper, first we show that the only function which satisfies fuzzy inner product space defined by Goudarzi and Vaezpour [6] is

$$F(x, y, t) = \begin{cases} 1 & , \quad t > 0 \\ 0 & , \quad t \leq 0. \end{cases}$$

* Corresponding author. Tel.: +98 391 3202355; fax: +98 391 3202270.
E-mail address: saheli@mail.vru.ac.ir (M. Saheli).

http://dx.doi.org/10.1016/j.fss.2015.11.008

Please cite this article in press as: M. Saheli, S. Khajepour Gelousalar, Fuzzy inner product spaces, Fuzzy Sets and Systems (2016), http://dx.doi.org/10.1016/j.fss.2015.11.008

In fact we must point out that there is no function that satisfies another fuzzy inner product space defined by Goudarzia and Vaezpour [6]. Moreover, the only functions that satisfy fuzzy inner product space defined by Mukherjee and Bag [10] are given in the following two forms:

$$F(x,y,t)=\begin{cases}1 & , \quad t>a\\ 0 & , \quad t\le a,\end{cases} \qquad \text{or} \qquad F(x,y,t)=\begin{cases}1 & , \quad t\ge a\\ 0 & , \quad t< a,\end{cases}$$

where $a \in R$.

As seen, only some trivial functions satisfy definitions of fuzzy inner product space which have been given in [6,10]. Therefore, all the results obtained in [6,10] are obvious. Hence, it is necessary to modify the definition of fuzzy inner product space in such a way that a non-trivial example of such a space can be given. This motivates us to weaken the conditions of fuzzy inner product space. Then, by an example, we show that every inner product space is a fuzzy inner product space. Moreover, there exists a nontrivial example for the new definition of fuzzy inner product space. Further, it is shown that our notion of fuzzy inner product space is the fuzzy normed linear space defined in [1]. In addition, we define a concept of orthogonality and study some properties of orthogonal elements in fuzzy inner product spaces. Finally, a relationship between the fuzzy inner product with the family of crisp inner products is studied, and it is demonstrated that any fuzzy inner product corresponds to a family of crisp inner products and vice versa.

2. Fuzzy Hilbert space

First, we define the fuzzy normed linear space given by Bag and Samanta.

Definition 2.1. (See [11].) A binary operation $* : [0,1] \times [0,1] \longrightarrow [0,1]$ is a t-norm if it satisfies the following for every $r, s, t, u \in [0,1]$:

(i) $1 * t = t$,
(ii) $t * s = s * t$,
(iii) $t * (r * s) = (t * r) * s$,
(iv) If $t \le s$ and $r \le u$ then $t * r \le s * u$.

Theorem 2.2. *(See [11].) If $*$ is a t-norm. Then*

$$t *_0 s \le t * s \le t *_1 s, \ \textit{for all}\ t, s \in [0,1],$$

where

$$t *_0 s = \begin{cases}\min\{t,s\} & , \quad \max\{t,s\}=1\\ 0 & , \quad \textit{otherwise},\end{cases}$$

and

$$t *_1 s = \min\{t,s\}.$$

Definition 2.3. (See [2].) A fuzzy normed space is a triplet $(X, N, *)$, where X is a real vector space, $*$ is a t-norm and N is a fuzzy set on $X \times R$ satisfying the following conditions for every $x, y \in X$ and $t, s \in R$:

(N1) $N(x,t) = 0$, for all $t \le 0$,
(N2) $N(x,t) = 1$, for all $t > 0$ if and only if $x = 0$,
(N3) $N(cx,t) = N(x, t/|c|)$, for all $(0 \ne) c \in R$,
(N4) $N(x,t) * N(y,s) \le N(x+y, t+s)$,
(N5) $N(x,.) : R \longrightarrow [0,1]$ is a non-decreasing function and $\lim_{t\to\infty} N(x,t) = 1$.

The $(X, N, *)$ will be referred to as a fuzzy normed linear space.