Available online at www.sciencedirect.com
ScienceDirect
ELSEVIER
Fuzzy Sets and Systems ••• (••••) •••–•••
FUZZY sets and systems
www.elsevier.com/locate/fss

Adaptive fuzzy tracking control for a class of pure-feedback stochastic nonlinear systems with non-lower triangular structure [☆]

Huanqing Wang [a,b,*], Xiaoping Liu [b], Kefu Liu [b]

[a] *School of Mathematics and Physics, Bohai University, Jinzhou 121000, Liaoning, PR China*
[b] *Faculty of Engineering, Lakehead University, Thunder Bay, ON, Canada*

Received 4 December 2014; received in revised form 25 August 2015; accepted 4 October 2015

Abstract

This paper considers the problem of the adaptive fuzzy tracking control for a class of nonaffine stochastic nonlinear systems without lower triangular form. By using fuzzy logic systems' universal approximation property, an adaptive fuzzy controller is proposed, which guarantees that all the signals in the closed-loop system are bounded in the sense of mean quartic value and the output tracking error eventually converges to a small neighborhood around the origin. Simulation results are provided to illustrate the effectiveness of the proposed scheme.

Keywords: Adaptive fuzzy control; Pure-feedback stochastic nonlinear systems; Backstepping

---

1. Introduction

During the past decades, many scholars have dedicated much effort to handle the stability analysis and control design for nonlinear systems and some interesting control algorithms have been proposed, such as adaptive control [24], backstepping control [16], fault tolerant control [41,42], and so on. Among them, adaptive backstepping control, a recursive design procedure, has become a powerful tool for controlling strict-feedback nonlinear systems with uncertain parameters, and many significant developments have been achieved, for example, see [7,16] for deterministic nonlinear systems and [10,15,20,21,37,38] for stochastic cases. To extend the applications of adaptive backstepping control, approximation-based adaptive fuzzy (or neural) control approaches were developed to deal with the tracking or regulation control problems of nonlinear systems with unknown functions. The main idea of the adaptive fuzzy or neural control methodology is that fuzzy-logic systems or neural networks are applied to approximate the unknown

---

[☆] This work is partially supported by the National Natural Science Foundation of China (61304002, 61333012 and 61304003), the Education Department of Liaoning Province under the general project research under Grant No. L2013424, and by the Natural Science Foundation of Liaoning Province of China (2015020058).

[*] Corresponding author.
E-mail address: ndwhq@163.com (H. Wang).

http://dx.doi.org/10.1016/j.fss.2015.10.001
0165-0114/© 2015 Elsevier B.V. All rights reserved.

nonlinearities in system dynamics, and the classical adaptive technique is used to construct controllers based on backstepping technique. Up to now, many valuable results on adaptive fuzzy or neural control for strict-feedback nonlinear systems have been reported in the literature [2–5,8,9,14,17–19,22,23,25–29,36,39,44,45] and the references therein.

The nonaffine pure-feedback systems representing a more general class of lower-triangular systems have no affine appearance of the variables to be used as virtual control signals or actual control signal, which makes the control of nonaffine pure-feedback nonlinear systems difficult and challenging. In practice, many control systems can be directly described by or transformed into non-affine structure, such as biochemical process, mechanical systems, Duffing oscillator, and so on [16]. In recent years, many researchers have paid much more attention to the controller design and stability analysis for nonaffine nonlinear systems [11,13,30,31,35,46]. In [13], by combining the backstepping methodology with adaptive neural design, several special cases of pure-feedback systems where the last one or two equations were assumed to be in affine form are investigated. Furthermore, some elegant control approaches were developed for completely pure-feedback systems [30] and other cases, such as pure-feedback systems with time-delay [35], deterministic disturbances [11]. More recently, approximation-based adaptive control of pure-feedback stochastic nonlinear systems has received increasing interest. In [43], an adaptive fuzzy tracking control scheme was proposed for a class of pure-feedback stochastic nonlinear systems which requires that the first $n-1$ subsystems are in affine form whereas the last differential equation is in non-affine form only. Thereafter, by combining adaptive backstepping together with fuzzy systems or neural networks, several control approaches were developed for state-feedback control in [6,32,33] and output-feedback control in [12] to completely pure-feedback stochastic nonlinear systems. Although a large amount of work has been carried out on the construction of backstepping-based adaptive controllers for pure-feedback stochastic nonlinear systems, all the aforementioned control approaches are only feasible under the condition that the considered systems are in lower-triangular structure. It is difficult to control the system when each subsystem function contains the whole system state variables. The main reason is that, during backstepping design, virtual control signal α_i which is designed in Step i to guarantee the stability of the first i subsystems must be the function of the current state vector $[x_1, \ldots, x_i]^T$ only. When the diffusion terms $\psi_i(.)$ in the ith subsystem is a nonlinear function of the whole state variables, how to design the virtual control signal α_i which is independent of the state variables x_j $(j = i+1, \ldots, n)$ is the main difficulty to be handled.

Based on the above observations, an adaptive fuzzy control method is presented for a class of nonaffine stochastic nonlinear systems via backstepping. It is proven that the proposed controller can guarantee that all the signals in the closed-loop system are bounded in the sense of mean quartic value, and the tracking error converges to a small neighborhood around the origin. The main contributions of this paper are twofold: i) a backstepping-based control methodology is systematically developed for a class of nonaffine stochastic nonlinear systems without lower triangular structure; ii) by estimating the maximum value of the norm of weight vector of fuzzy systems, only one adaptive parameter is needed for an n-order nonlinear systems. In this way, the computational burden is alleviated significantly, which might render this control design more suitable for practical applications.

The remainder of this paper is organized as follows. The problem formulation and preliminaries are given in Section 2. An adaptive fuzzy control scheme is presented in Section 3. The simulation examples are given in Section 4, and followed by Section 5 which concludes the work.

2. Problem formulation and preliminaries

In this paper, we consider a class of stochastic pure-feedback nonlinear system in the following form:

$$\begin{cases} dx_i = f_i(\bar{x}_i, x_{i+1})dt + \psi_i^T(x)dw, 1 \le i \le n-1, \\ dx_n = f_n(\bar{x}_n, u)dt + \psi_n^T(x)dw, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $u \in R$ and $y \in R$ are the state variable, system input, and system output, respectively, $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T \in R^i$, w is an r-dimensional standard Brownian motion defined on the complete probability space $(\Omega, F, \{F_t\}_{t\ge 0}, P)$ with Ω being a sample space, F being a σ-field, $\{F_t\}_{t\ge 0}$ being a filtration, and P being a probability measure. The drifting terms $f_i(\cdot) : R^{i+1} \to R$ and the diffusion terms $\psi_i(\cdot) : R^n \to R^r$, $(i = 1, 2, \ldots, n)$ are unknown smooth nonlinear functions.

Please cite this article in press as: H. Wang et al., Adaptive fuzzy tracking control for a class of pure-feedback stochastic nonlinear systems with non-lower triangular structure, Fuzzy Sets and Systems (2015), http://dx.doi.org/10.1016/j.fss.2015.10.001