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Logical characterizations of simulation and bisimulation for fuzzy transition systems

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Abstract

Simulations and bisimulations are known to be useful for abstracting and comparing formal systems, and they have recently been introduced into fuzzy systems. In this study, we provide sound and complete logical characterizations for simulation and bisimulation, which are defined over fuzzy labeled transition systems via two variants of the Hennessy–Milner Logic. The logic for characterizing fuzzy simulation has neither negation nor disjunction, which is very different from the well-known logical characterizations of probabilistic simulations, although the completeness proofs of our characterization results are inspired by relevant research in probabilistic concurrency theory. The logic for characterizing fuzzy bisimulation also deviates from that for probabilistic bisimulations.

Keywords: Bisimulation; Completeness; Fuzzy labeled transition system; Logical characterization; Simulation; Soundness

1. Introduction

The analysis of fuzzy systems has been the subject of active research during the last 60 years and many formalisms have been proposed for modeling them, including fuzzy automata (e.g., see [2,3,7,27,28,30,35,37,43]), fuzzy Petri nets [39], fuzzy Markov processes [4], and fuzzy discrete event systems [29,36,38].

Recently, a new formal model for fuzzy systems called fuzzy labeled transition systems (**FLTS**s) was proposed [6,17,23]. **FLTS**s are a natural generalization of the classical labeled transition systems in computer science, where after performing some action, a system evolves from one state into a fuzzy set of successor states instead of a unique state. Many formal description tools for fuzzy systems, such as fuzzy Petri nets and fuzzy discrete event systems [29,36], are not **FLTS**s. However, it is possible to translate a system's description in one of these formalisms into an **FLTS** to represent its behavior.

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Bisimulation [33] has been investigated in depth in process algebras because it offers a convenient co-inductive proof technique for establishing behavioral equivalence [31]. Bisimulation has mostly been used for verifying formal systems and it is the foundation of state-aggregation algorithms, which compress models by merging bisimilar states. State aggregation is used routinely as a preprocessing step before model checking [1,18]. Recently, bisimulation has been introduced into fuzzy systems. For example, Cao et al. [6] considered bisimulations for **FLTS**s. Bisimulation-based reasoning also appeared for fuzzy automata and fuzzy discrete event systems [10,11,17,32,41,44].

Following a seminal study that explored the connection between bisimulation and modal logic [21], many studies have characterized various types of bisimulations using appropriate logics, e.g., [15,16,22,26]. A logic characterizes a bisimulation soundly and completely when two states are bisimilar if and only if they satisfy the same set of logical formulae. The significance of logical characterizations is twofold. Based on a sound and complete logical characterization, the problem of checking whether two states are bisimilar is converted into a logical judgment of whether two states satisfy the same set of logical formulae, which can benefit from traditional logic theories and be assisted by some practical tools. A logical characterization also allows model checking to be performed based on a bisimulation quotient transition system because a logical formula holds for the quotient if and only if it holds for the original transition system.

In the present study, we provide logical characterizations of bisimulation and simulation for **FLTS**s. Often, a state or system can simulate another but not *vice versa*. For example, when we check that an implementation matches its specification, we normally do not demand that the implementation performs anything more than that required. It is usually acceptable that the implementation simulates its specification. Hence, it is also interesting to investigate simulations. Unlike other studies of fuzzy systems, we define simulation and bisimulation by virtue of closed subsets of some binary relation (Section 5 provides a detailed discussion). Moreover, some recent studies of **FLTS**s and fuzzy automata have focused mainly on simulations and bisimulations [6,10,13,25,41], whereas they did not consider logical characterizations. A logical characterization of fuzzy bisimulation was provided by [17], but the differences from the current study are as follows: (1) the logic used in [17] employs recursive formulae where it interprets a formula as a fuzzy set that gives the measure of satisfaction and unsatisfaction for the formula; and (2) we consider bisimulation and simulation, whereas [17] only considered bisimulation.

The logic used to characterize fuzzy bisimulation is very similar to Larsen and Skou's probabilistic extension of the Hennessy–Milner Logic.[1] The completeness proof for our logical characterization of fuzzy bisimulation is also inspired by [22], who characterized probabilistic bisimulation. Indeed, there is only a slight difference between the **FLTS** model and probabilistic labeled transition systems (**PLTS**s). This may create the impression that the current study is a straightforward generalization of the study of **PLTS**s, but this is not the case. For **PLTS**s, disjunction is necessary to characterize simulation, whereas it is not for **FLTS**s. For **PLTS**s, negation is not necessary for characterizing bisimulation and binary conjunction is already sufficient, whereas for **FLTS**s, both negation and infinite conjunction are necessary to characterize bisimulation for general **FLTS**s, which may be infinitely branching. Moreover, different techniques are needed to prove characterization theorems for **FLTS**s and **PLTS**s. For example, in the case of **PLTS**s, the well-known π–λ theorem holds, which greatly simplifies the completeness proof for the logical characterization of bisimulation. However, the π–λ theorem is invalid for **FLTS**s, so we adopt a different approach to prove completeness, where we try to construct a characteristic formula for each equivalence class, i.e., the formula is satisfied only by the states in that equivalence class. Sections 4.2 and 4.3 provide more details.

The remainder of this paper is organized as follows. We briefly review some of the basic concepts used in this study in Section 2. In Section 3, we describe some properties of simulations and bisimulations for **FLTS**s. In particular, similarity and bisimilarity are shown to be closed under the parallel composition of **FLTS**s. Section 4 presents the logical characterization theorems. In this section, we also analyze why the logics characterizing bisimulations for **FLTS**s and **PLTS**s are different. We introduce some related research in Section 5. Finally, we give our conclusions in Section 6 by providing a summary of the differences between the logical characterizations of **FLTS**s and **PLTS**s, as well as discussing future research.

[1] Our logic for fuzzy systems originates from computer science and it is intended to be used for reasoning about fuzzy labeled transition systems, and thus it differs from the fuzzy logic investigated in [40].

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