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Further improvements of determinization methods for fuzzy finite automata ☆

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Abstract

In this paper we provide further improvements of determinization methods for fuzzy finite automata. These methods perform better than all previous determinization methods for fuzzy finite automata, developed by Bělohlávek [3], Li and Pedrycz [38], Ignjatović et al. [25], and Jančić et al. [33], in the sense that they produce smaller automata, while require the same computation time. The only exception is the Brzozowski type determinization algorithm developed recently by Jančić and Ćirić [34], which produces a minimal crisp-deterministic fuzzy automaton, but the algorithms created here can also be used within the Brzozowski type algorithm and improve its performance.

Keywords: Fuzzy automaton; Crisp-deterministic fuzzy automaton; Nerode automaton; Determinization; State reduction; Complete residuated lattice

1. Introduction

Many practical applications of automata require determinization, a procedure of converting a nondeterministic finite automaton to an equivalent deterministic finite automaton, or, in the case of fuzzy automata, a procedure of converting a fuzzy finite automaton to an equivalent crisp-deterministic fuzzy automaton. The standard determinization method is the subset construction, where a nondeterministic automaton with n states is converted to an equivalent deterministic automaton with up to 2^n states, whereas in the case of fuzzy finite automata the resulting crisp-deterministic fuzzy automaton can even be infinite. That is why the main research directions in this area are aimed at finding such methods which will mitigate the potential enormous growth of the number of states during the determinization. The natural idea is to combine the existing determinization and state reduction methods so that we reduce

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the number of states to determinization. However, here we combine these methods to provide two-in-one procedures that perform determinization and state reduction simultaneously.

A crisp-deterministic fuzzy automaton is a fuzzy automaton with exactly one crisp initial state and a deterministic transition function, and the fuzziness is entirely concentrated in the fuzzy set of terminal states. This kind of determinism was first studied by Bělohlávek [3], in the context of fuzzy finite automata over a complete distributive lattice, and Li and Pedrycz [38], in the context of fuzzy finite automata over a lattice-ordered monoid. Determinization algorithms that were provided there generalize the subset construction. Another algorithm, provided by Ignjatović et al. [25], also generalizes the subset construction, and for any input it generates a smaller crisp-deterministic fuzzy automaton than the algorithms from [3,38]. Since this crisp-deterministic fuzzy automaton can be alternatively constructed by means of the Nerode right congruence of the original fuzzy finite automaton, it was called in [27] the *Nerode automaton* of this fuzzy finite automaton. The Nerode automaton was constructed in [25] for fuzzy finite automata over a complete residuated lattice, and it was noted that the identical construction can also be applied in a more general context, for fuzzy finite automata over a lattice-ordered monoid, and even for weighted finite automata over a semiring. This construction was also transferred in [16] to weighted automata over strong bimonoids. The algorithm proposed by Jančić et al. in [33] produces a crisp-deterministic fuzzy automaton that is even smaller than the Nerode automaton. In the terminology introduced in this paper, Jančić et al. constructed the children automaton for the Nerode automaton of a given fuzzy finite automaton. Recently, Jančić and Ćirić [34] adapted the well-known Brzozowski's double reversal determinization algorithm to fuzzy automata. As in the case of ordinary nondeterministic automata, Brzozowski type determinization of a fuzzy finite automaton results in a minimal crisp-deterministic fuzzy automaton that is equivalent to the original fuzzy finite automaton. It was also shown that even if all previous determinization algorithms fail to build a finite crisp-deterministic fuzzy automaton, the Brzozowski type algorithm can produce a finite one.

In addition to the determinization, practical applications of automata often require state reduction, a procedure of converting a given automaton into an equivalent automaton with a smaller number of states. As the state minimization problem for fuzzy finite automata, as well as for nondeterministic ones, is computationally hard (PSPACE-complete [35,37,69]), it is not required that this equivalent automaton is minimal, but it is necessary that it is effectively computable. From different aspects, the state reduction for fuzzy automata was studied in [1,15,36,44,48,50,51,64,67], as well as in the books [46,49]. All algorithms provided there were motivated by the basic idea used in the minimization of ordinary de terministic automata, the idea of detecting and merging indistinguishable states, which boils down to computation of certain crisp equivalences on the set of states. A new approach to the state reduction was initiated in [20,59]. First, it was shown that better reductions of fuzzy finite automata can be achieved if fuzzy equivalences are used instead of ordinary equivalences, and even better if fuzzy quasi-orders are used. In addition, it was shown that the state reduction problem for fuzzy finite automata can be reduced to the problem of finding fuzzy quasi-orders that are solutions to a particular system of fuzzy relation equations, called the general system. As the general system is difficult to solve, the problem was further reduced to the search for instances of the general system and their solutions which ensure the best possible reductions and can be efficiently computed. Two such instances, whose solutions were called right and left invariant, have the greatest solutions that can be computed in polynomial time, and two others, whose solutions were called weakly right and left invariant, have the greatest solutions that ensure better reductions, but their computation requires exponential time. For information about the reduction of the number of states of ordinary nondeterministic automata using right and left invariant equivalences and quasi-orders we refer to articles [9,10,28–31], as well as to articles [17,20,59] where relationships between the state reduction of fuzzy and nondeterministic automata are explained in detail.

The main aim of this paper is to combine determinization and state reduction methods into two-in-one algorithms that simultaneously perform determinization and state reduction. These algorithms perform better than all previous determinization algorithms for fuzzy finite automata, developed in [3,25,33,38], in the sense that they produce smaller automata, while require the same computation time. The only exception is the Brzozowski type determinization algorithm developed recently in [34], which produces a minimal crisp-deterministic fuzzy automaton, but we will see that the algorithms created here can be used within the Brzozowski type algorithm and improve its performance.

Our main results are the following. For any fuzzy finite automaton $\mathcal{A}$ and a reflexive weakly right invariant fuzzy relation φ on $\mathcal{A}$, we construct a crisp-deterministic fuzzy automaton $\mathcal{A}_\varphi$ and prove that it is equivalent to $\mathcal{A}$. If φ is a weakly right invariant fuzzy quasi-order, we show that the same automaton $\mathcal{A}_\varphi$ would be produced if we first perform the state reduction of $\mathcal{A}$ by means of φ and then construct the Nerode automaton of this reduced automaton. Furthermore, we show that automata $\mathcal{A}_\varphi$ determined by right invariant fuzzy quasi-orders are smaller than the Nerode