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Capacities and overlap indexes with an application in fuzzy rule-based classification systems

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Abstract

In this work, we introduce a method for constructing capacities using overlap indexes between the fuzzy sets which are generated from the inputs of the considered problem. We also use these capacities to aggregate information by means of the Choquet integral in a fuzzy rule-based classifier. We observe that with these capacities the obtained results are better than those obtained with other measures.

Keywords: Capacity; Overlap index; Overlap function; Choquet integral; Fuzzy rule-based classification systems

1. Introduction

For some problems, the solution may be improved in two different ways:

(a) employing representations of information that take the inherent imprecision in data into account [19,22,39], and
(b) using information fusion mechanisms adapted to such representations [6,7,11].

These strategies were, for instance, applied to the calculation of the volume of clinically significant regions in the brain [50] and to the biometric recognition from digital fingerprints [40].

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In many problems, it is crucial to find a relation between groups of data. Such relation can be expressed, for instance, in terms of an appropriate fuzzy measure or capacity that is capable to detect the links between data [43].

In this work, we focus on the latter problem. In particular, our main goal is to describe a method to build capacities [43,49] from the data (inputs) of a given problem.

The main idea is as follows: after constructing fuzzy sets from the data (inputs) of the problem, we establish links between these fuzzy sets in terms of overlap indexes [8,19,25] that are derived from overlap functions [10,7,34]. Finally, capacities are defined on the basis of these overlaps.

The notion of an overlap function appears in image processing and classification settings as a way to measure to what extent a given element belongs to two (in the original bidimensional case) or three or more (in the multidimensional case) considered classes. The interest of using bidimensional overlap functions for classification was made clear in [7], where they were used to define an analog of preference structures in those situations where associativity is not required or even natural. Moreover, they can also be used for approximate reasoning, as shown in [25]. Basically, a bidimensional overlap function is a continuous aggregation function which vanishes whenever any of the inputs is equal to zero and which equals one only when both inputs are 1. In particular, continuous t-norms without divisors of zero are examples of overlap functions [10].

These capacities are used for the aggregation of information through the use of Choquet integrals. Note that, in this way, the resulting aggregation function can be adapted to the specific problem at hand. To illustrate our approach, we apply interpretable Fuzzy Rule-Based Classification Systems (FRBCSs) [32] to some benchmark classification problems [20]. In this setting, the aggregation of the information plays a key role since it determines the character of the Fuzzy Reasoning Method (FRM) [15].

Specifically, we apply the Choquet integral to aggregate the local information given by each fuzzy rule of the system. The fuzzy sets used to construct the associated fuzzy measure are the rule weights and consequently, it expresses the interaction among the rules of the different classes. Furthermore, we propose an evolutionary method to learn a different capacity for each class of the problem. The quality of the proposal is tested using 40 datasets selected from the KEEL dataset repository [1]. We compare our approach with 6 classifiers, namely SGERD [41], SLAVE [28], C4.5 [44], CART [5], RIPPER [14] and FURIA [30], and we use statistical tests to support our conclusions.

This paper is organized as follows: after providing some preliminaries, we analyse some properties of overlap functions and indexes. In Sections 4 and 5, we present a method for constructing capacities from overlap indexes. In Sections 6 and 7, we study the main properties of our constructions. Section 8 presents the FRM in which we apply Choquet integrals defined in terms of our measures to FRBCSs. Section 9 exhibits both the results obtained by our approach in a set of well-known classification problems and the statistical analysis. Finally, we present some conclusions, future research directions and references.

2. Preliminaries

Given a set U, we denote its cardinality by $card(U)$.

Recall that, given a referential set (or universe) U, a fuzzy set A over U corresponds to a function μ_A:

$$\mu_A : U \to [0, 1] .$$

More precisely, a fuzzy set A is given by the graph of the function μ_A, called the membership function of A:

$$\{(i, \mu_A(i)) \mid i \in U\} .$$

For simplicity, we write $A(i)$ instead of $\mu_A(i)$ in this work.

Given a referential set U, we denote by $FS(U)$ the space of all fuzzy sets defined over U. In this work, we only deal with finite referential sets. Henceforth, let $U = \{1, \dots, n\}$.

$FS(U)$ can be endowed with a partial order $\subseteq$ as follows. For $A, B \in FS(U)$, $A \subseteq B$ if and only if $A(i) \le B(i)$ for every $i \in U$. Similarly, a partial order on $[0, 1]^n$ is given as follows: $(x_1, \dots, x_n) \le (y_1, \dots, y_n)$ if and only if $x_i \le y_i$ for every $i \in U$. Together with these partial orders, $FS(U)$ and $[0, 1]^n$ constitute lattices, that is, all pair-wise infima and suprema exist in $FS(U)$ and $[0, 1]^n$, respectively.

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