Information Sciences 424 (2018) 224–234

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Exponential stability and $\mathcal{L}_1$-gain analysis for positive time-delay Markovian jump systems with switching transition rates subject to average dwell time

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ARTICLE INFO

Article history:
Received 23 November 2016
Revised 25 September 2017
Accepted 3 October 2017
Available online 3 October 2017

Keywords:
Exponential stability
Switching transition rates
Linear co-positive Lyapunov function
Average dwell time

ABSTRACT

The paper deals with the problems of exponential stability and $\mathcal{L}_1$-gain analysis for positive time-delay Markovian jump systems (MJSs) with switching transition rates. Another set of useful regime-switching model is given, which extends fixed transition rates to time-varying transition rates. By resorting to the linear co-positive Lyapunov function and average dwell time, sufficient conditions for exponential stability are proposed in terms of standard linear programming. Based on the obtained results, $\mathcal{L}_1$-gain performance is analyzed. Finally, an example is proposed to illustrate the validity of the main results.

1. Introduction

In practical systems, there exists a special class of dynamic systems whose state variables and output signals remain nonnegative whenever both the initial condition and the input signal are nonnegative. This unordinary kind of systems, usually denoted as positive systems [8,12], also known as nonnegative systems, have extensive applications including industrial engineering [3] and communication networks [31]. Over the past decade, positive systems have received ever-increasing research interest and many interesting properties have been unraveled (see e.g., [1,4,6,10,23,33,36,45]). For example, the l_1-induced norm has been constructed and the existence of l_1 state feedback controller has been solved by using an iterative convex optimization approach in [4]. For continuous-time positive switched linear systems [33], sufficient conditions for asymptotic stability under appropriate switching have been given by constructing the joint linear co-positive Lyapunov function. For positive Takagi-Sugeno fuzzy nonlinear systems [45], some less conservative conditions for stability have been proposed and a constrained fuzzy tracking controller has been designed to ensure the tracking performance and positivity of closed-loop system.

At the same time, time delay [48,49] is very common in practical dynamical systems, such as manufacturing systems, networked control systems, economic systems, biological systems, telecommunication and chemical systems, etc. It has been shown that time delay is usually an important factor of instability or poor performance in a dynamical system. Recently,

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https://doi.org/10.1016/j.ins.2017.10.008
0020-0255/

many significant results on stability, stabilization, and performance analysis for positive systems with time delay can be found (see e.g., [7,11,22,30,37,38,43]). For example, for positive two-dimensional Takagi-Sugeno fuzzy systems with state delays [7], the l_1-gain performance analysis has been studied. For fuzzy positive systems with time-varying delays [11], the $\mathcal{L}_-$ fault detection filter and multi-objective $\mathcal{L}_-/\mathcal{L}_1$ fault detection filter design problems have been addressed. By constructing the Lyapunov–Krasovskii functional, an absolute exponential $\mathcal{L}_1$ controller for positive switched nonlinear time-delay systems has been designed in [38]. The $\mathcal{H}_\infty$ dynamic output feedback controller gains for positive systems with multiple delays have been obtained by use of cone complementarity linearization techniques in [43].

On the other hand, as a special class of hybrid systems, MJSs have some advantages of describing dynamic systems subject to sudden changes between subsystems caused by external causes, such as networked control systems, manufacturing systems, fault-detection systems, and system theory (see e.g., [13–19,26–29,32,34,35,41,42,44]). To mention a few, for uncertain time-delay systems with Markovian switching parameters [13], some new sufficient conditions for delay-dependent stochastic stability and stabilization with $\mathcal{H}_\infty$ performance have been established in terms of linear matrix inequalities. In [14], a sliding-mode approach has been proposed for the exponential $\mathcal{H}_\infty$ synchronization problem of master-slave time-delay systems with nonlinear uncertainties and Markovian jumping parameters. In [28], for continuous-time Markovian jump linear systems with deficient transition descriptions, the authors have developed an input-output approach to the delay-dependent stability analysis and $\mathcal{H}_\infty$ controller synthesis. For continuous-time Markovian jump linear systems with time-varying delay and partially accessible mode information [34], by utilizing the model transformation idea and the scaled small gain theorem, the underlying full-order and reduced-order $\mathcal{H}_\infty$ filtering synthesis problems have been formulated. For discrete-time Markovian jump nonlinear systems with time-delays represented by Takagi–Sugeno model [44], sufficient conditions in the form of linear matrix inequalities have been presented for stochastic finite-time $\mathcal{H}_\infty$ stabilization via observer-based fuzzy state feedback. However, many results about positive MJSs [2,9,20,21,24,25,39,40,46] have been obtained in view of time-invariant transition rates. In fact, different from those uncertain or partly known transition rates, an extension to Markovian jump model with time-varying transition rates can provide another set of useful regime-switching model. In such model, MJSs with switching transition rates have some advantages over that with fixed transition rates in terms of flexibility [47]. Recently, some papers concerning MJSs with switching transition rates [5,47] have been published.

This paper will deal with the issues of exponential stability and $\mathcal{L}_1$-gain analysis for positive MJSs with time-varying delay and switching transition rates. It is necessary to point out the differences between the present work and the existing relative works [2,9,20,21,24,25,37,39,40,46]. First, the switching law in [37] followed asynchronous switching law with mode-dependent average dwell time while the switching law in this paper follows the Markovian process with time-varying property. Second, the literatures [2,9,20,21,24,25,39,40,46] mainly investigated the positive MJSs with time-invariant transition rates while the transition rates in this paper are time-varying. Third, the literatures [2,9,24,39,46] mainly investigated the positive MJSs without time delay while the model in this paper considers time delay. The problems of exponential stability and $\mathcal{L}_1$-gain analysis for positive MJSs with time-varying delay and switching transition rates are theoretically challenging, difficult, and open all the time. More specifically, compared with positive time-delay MJSs with time-invariant transition rates, it is very difficult to handle positive time-delay MJSs with time-varying transition rates. Moreover, one needs to consider not only stability of dynamic systems governed by the Markovian process, but also the constrained positivity. In addition, the dynamical behaviors are affected by the interaction among Markovian process, time delay, switching transition rates, disturbance input, and positivity, which are very complicated. These above aspects motivate our current research work. The main contributions are given as follows: (i) By resorting to the linear co-positive Lyapunov function and average dwell time, sufficient conditions for exponential stability are proposed; (ii) The disturbance attenuation performance (the $\mathcal{L}_1$-gain performance) is analyzed; (iii) Additionally, due to the quantity of virus mutation treatment model satisfying the nonnegative property and abrupt change of genetic mutation, modeling virus mutation treatment model as positive MJSs shows the validity of the main algorithms.

Notation. $A\succeq$ ($\preceq 0$, $\succ$, $\prec$) means all entries of matrix A are nonnegative (non-positive, positive, negative); $A\succ B$ ($A\succeq B$) means $A-B\succ 0$ ($A-B\succeq 0$). $\mathbb{R}$ ($\mathbb{R}_+$) denotes the set of all real (positive real) numbers; $\mathbb{R}^n$ ($\mathbb{R}^n_+$) stands for n-dimensional real (positive) vector space. The vector 1-norm is denoted by $||x||_1=\sum_{k=1}^n |x_k|$, where x_k is the kth element of $x\in\mathbb{R}^n$; $A=[a_{ij}]_{n\times n}$ is a Metzler matrix if $a_{ij}\geq 0$, $i\neq j$. Symbol $E\{\cdot\}$ stands for the mathematical expectation. $\mathbf{1}_n$ means the all-ones vector in $\mathbb{R}^n$.

2. Problem statement and preliminaries

Consider the positive time-delay MJSs as follows:

$$\begin{aligned}\dot{x}(t) &= A(g_t)x(t)+A_d(g_t)x(t-d(t))+G(g_t)w(t),\\ z(t) &= C(g_t)x(t)+D(g_t)w(t),\\ x(t_0+\theta) &= \varphi(\theta), \forall\theta\in[-d,0],\end{aligned} \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $w(t)\in\mathbb{R}^l$, and $z(t)\in\mathbb{R}^q$ are, respectively, the state vector, the disturbance input, and the estimated output; $\varphi(\theta)$ is the initial condition; g_t stands for a time-homogeneous Markovian process and takes values in $S_1=\{1,2,\dots,N\}$ with