Information Sciences 424 (2018) 385–399

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

New types of aggregation functions for interval-valued fuzzy setting and preservation of pos-*B* and nec-*B*-transitivity in decision making problems

Urszula Bentkowska

Faculty of Mathematics and Natural Sciences, Interdisciplinary Centre for Computational Modelling, University of Rzeszów, Rzeszów, Poland

ARTICLE INFO

Article history:
Received 23 February 2017
Revised 20 September 2017
Accepted 8 October 2017
Available online 10 October 2017

MSC:
03E72
26E60
68T37

Keywords:
Aggregation function
Pre-aggregation function
Partial order
Interval order
B-transitivity
Pos-*B*-transitivity
Nec-*B*-transitivity
Interval-valued fuzzy relation
Decision making

ABSTRACT

In this contribution new types of aggregation functions for interval-valued fuzzy setting are introduced. They are called necessary and possible aggregation functions, respectively. In the monotonicity conditions for these aggregation functions the classical monotonicity for intervals is replaced with the new comparability relations. These relations follow naturally from the interpretations of interval-valued fuzzy sets and together with the classically used monotonicity for intervals, form a family of all possible approaches to define relations of comparability for intervals. Moreover, in this paper dependencies between considered families of aggregation functions are presented. Furthermore, transitivity properties of interval-valued fuzzy relations, based on these new comparability relations, are studied and preservation of them by possible and necessary aggregation functions are considered.

1. Introduction

Interval-valued fuzzy sets were introduced by Zadeh [33] as a generalization of the concept of a fuzzy set [32]. Interval valued fuzzy sets and relations have applications in diverse types of areas, for example in classification, image processing and multicriteria decision making.

In [20], a comparative study of the existing definitions of order relations between intervals, analyzing the level of acceptability and shortcomings from different points of view were presented. Comparability relations used in the interval-valued setting may be connected with ontic and epistemic interpretation of intervals [11,12]. Epistemic uncertainty represents the idea of partial or incomplete information. Simply, it may be described by means of a set of possible values of some quantity of interest, one of which is the right one. A fuzzy set represents in such approach incomplete information, so it may be

E-mail address: ududziak@ur.edu.pl

https://doi.org/10.1016/j.ins.2017.10.025
0020-0255/

called disjunctive [11]. On the other hand, fuzzy sets may be conjunctive and can be called ontic fuzzy sets [11]. In this situation the fuzzy set is used as representing some precise gradual entity consisting of a collection of items.

The aim of this work is to introduce new types of aggregation functions for interval-valued fuzzy setting. In the monotonicity condition of an aggregation function the natural (partial) order, from the family of intervals, is replaced with new comparability relations. These relations are really new comparability relations, since they have other properties than the natural order has. These relations follow naturally from the epistemic setting of interval-valued fuzzy sets and together with the natural order used in this setting form a family of all possible approaches to define comparability relations for intervals. The origin of these relations is other than the one presented in [3] for partial order used classically in interval-valued fuzzy setting.

Moreover, in this paper dependencies between the new introduced families of aggregation functions and known families of aggregation functions are studied for interval-valued fuzzy setting. Many examples of new types of aggregation functions are presented, including proper aggregation functions of the new type, i.e. not coinciding with the traditional families of aggregation functions. For this purpose, pre-aggregation functions turn out to be a useful tool. Characterization of one of the new families of aggregation functions is given in the case of decomposability. The important fact is that these new families of aggregation functions cannot be translated as a special case of aggregation functions on lattice. This follows from the fact that the family of intervals with each of the relation is not a partially ordered set, so it is not a lattice. Moreover, in this paper preservation of properties of interval-valued fuzzy relations by the new type of aggregation functions is discussed. These properties also follow from the interpretation of the new 'comparability' relations for the epistemic setting of fuzzy relations. We consider here only transitivity property which is one of the most interesting properties of relations. Other properties may be found in [26].

These considerations have possible applications in multicriteria (or similarly multiagent) decision making problems with intervals (not just numbers in [0, 1]). In virtue of using diverse approaches of defining the comparability relations for the intervals we may have applications depending on the presented problem from real-life situations. In such cases it may be interesting to use for aggregation of the given data (gathered as interval-valued fuzzy relations) adequate type of an aggregation function, which follows from the assumed interpretation.

The paper is structured as follows. Firstly, some concepts and results useful in further considerations are given (Section 2). Next, new types of aggregation functions for intervals are introduced and dependencies between them are studied (Section 3). Moreover, aggregation (by new types of aggregation functions) of interval-valued fuzzy relations having pos-B-transitivity and nec-B-transitivity are studied (Section 4). To finish, in Section 5 an algorithm presenting possible application is presented.

2. Interval-valued fuzzy relations

First, we recall definition of the lattice operations and the classical order for interval-valued fuzzy relations. We consider relations instead of sets, since we will concentrate ourselves on interval-valued fuzzy relations and their properties. Let X, Y, Z be non-empty sets.

Definition 1 (cf. [27,33])**.** An interval-valued fuzzy relation R between universes X, Y is a mapping R: $X\times Y\to L^I$ such that

$$R(x,y)=[\underline{R}(x,y),\overline{R}(x,y)]\in L^I,$$

for all couples $(x, y)\in(X\times Y)$, where $L^I=\{[x_1,x_2]:\ x_1,x_2\in[0,1],\ x_1\leqslant x_2\}$.

The class of all interval-valued fuzzy relations between universes X, Y will be denoted by $\mathcal{IVFR}(X\times Y)$ or $\mathcal{IVFR}(X)$ for $X=Y$. The well-known classical monotonicity (partial order) for intervals is of the form

$$[x_1,y_1]\preceq[x_2,y_2]\Leftrightarrow x_1\leqslant x_2, y_1\leqslant y_2. \tag{1}$$

$\mathcal{IVFR}(X\times Y)$ with $\preceq$ is partially ordered and moreover it is a lattice. We will consider other comparability relations on L^I as a consequence on $\mathcal{IVFR}(X\times Y)$. To begin with, we recall the definition of an interval order [14–16] for crisp relations.

Definition 2 ([17], p. 42)**.** A relation $R\subset X\times X$ is an interval order if it is complete and has the Ferrers property, i.e.:
$R(x, y)$ or $R(y, x)$, for $x, y\in X$,
$R(x, y)$ and $R(z, w)\Rightarrow R(x, w)$ or $R(z, y)$, for $x, y, z, w\in X$, respectively.

Now we consider the following comparability relations on L^I (cf. [25]):

$$[x_1,y_1]\preceq_\pi[x_2,y_2]\Leftrightarrow x_1\leqslant y_2, \tag{2}$$

$$[x_1,y_1]\preceq_\nu[x_2,y_2]\Leftrightarrow y_1\leqslant x_2. \tag{3}$$

These relations, including classical order, follow from the epistemic setting of interval-valued fuzzy relations and form the full possible set of interpretations of comparability relations on intervals. Detailed discussion on this subject is presented in [26] (cf. [25]). We will only briefly recall the interpretation of conditions (2) and (3). Relation (2) follows from the following interpretation of inclusion: there exists an instance in the left interval and there exists an instance in the right interval