Information Sciences 423 (2018) 1–23

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A tri-objective differential evolution approach for multimodal optimization

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ARTICLE INFO

Article history:
Received 28 March 2017
Revised 11 September 2017
Accepted 15 September 2017
Available online 18 September 2017

Keywords:
Multimodal optimization problems
Multiobjective optimization
Differential evolution
Niching method

ABSTRACT

The multimodal optimization problems (MMOPs) need to find multiple optima simultaneously, so the population diversity is a critical issue that should be considered in designing an evolutionary optimization algorithm for MMOPs. Taking advantage of evolutionary multiobjective optimization in maintaining good population diversity, this paper proposes a tri-objective differential evolution (DE) approach to solve MMOPs. Given an MMOP, we first transform it into a tri-objective optimization problem (TOP). The three optimization objectives are constructed based on 1) the objective function of an MMOP, 2) the individual distance information measured by a set of reference points, and 3) the shared fitness based on niching technique. The first two objectives are mutually conflicting so that the advantage of evolutionary multiobjective optimization can be fully used. The population diversity is greatly improved by the third objective constructed by the niching technique which is insensitive to niching parameters. Mathematical proofs are given to demonstrate that the Pareto-optimal front of the TOP contains all global optima of the MMOP. Subsequently, DE-based multiobjective optimization techniques are applied to solve the converted TOP. Moreover, a modified solution comparison criterion and an adaptive ranking strategy for DE are introduced to improve the accuracy of solutions. Experiments have been conducted on 44 benchmark functions to evaluate the performance of the proposed approach. The results show that the proposed approach achieves competitive performance compared with several state-of-the-art multimodal optimization algorithms.

1. Introduction

Many real-world applications involve optimization problems with multiple (equal) global optima and require to find all the optima simultaneously. This kind of problems is known as multimodal optimization problems (MMOPs) [3,50]. For example, in engineering areas, for a given nonlinear equation system, problem solvers should find out as many optimal solutions as possible to enable the decision-maker make decisions according to his/her preference [42]. However, different from the optimization problems which require finding only one global optimum [6,49], it is more challenging to locate multiple optima, simultaneously.

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http://dx.doi.org/10.1016/j.ins.2017.09.044

Although specialized in solving optimization problems, the classical evolutionary algorithms (EAs) such as genetic algorithm (GA) [15], differential evolution (DE) [39], and particle swarm optimization (PSO) [21] cannot be directly applied to MMOPs. This is because the evolutionary operators guide the population to converge toward one prominent solution while ignoring the others.

To enable EAs for solving MMOPs, the niching techniques are required to improve the population diversity and to prevent all individuals from converging to the first discovered optimum. The well-known niching methods include crowding [41], clearing [28], restricted tournament selection [18], fitness sharing [27], and speciation [23]. However, their performance is highly sensitive to niching parameters, such as the parameters controlling the niching radius. Therefore, designing parameter-free or parameter insensitive techniques have become a key issue in the study of niching methods [2,4,12,22,32,33,45,46].

Recently, some relevant works [1,8,43,47] have been proposed to convert an MMOP to a multiobjective optimization problem (MOP) and then solve the MOP. The advantages are discussed as follows. On the one hand, they avoid the problem-dependent niching parameters. On the other hand, by transforming an MMOP into an MOP, it is now available to use various multiobjective optimization techniques to produce multiple optimal solutions directly without introducing auxiliary methods such as local search, archive technique or clustering, which makes the algorithm simple and easy to follow. However, it is not a trivial work to convert an MMOP to an MOP. Since objective confliction is the prerequisite for multiobjective optimization [43], if the optimization objectives of the transformed MOP do not conflict or conflict weakly with each other, the multiobjective optimization techniques will not work. To address this issue, this paper proposes a novel conversion to transform an MMOP into a tri-objective optimization problem (TOP). The first two objectives are constructed mutually conflicting, so that the advantage of evolutionary multiobjective optimization can be fully used. Further, instead of using clustering techniques to maintain population diversity [1,8], the distance information among individuals and fitness sharing based on niching are utilized to construct the third objective to enhance population diversity in our proposed approach. Accordingly, a tri-objective DE algorithm for multimodal optimization named TriDEMO, which is an extension of our previous work [20], has been developed. The main characteristics of TriDEMO are summarized as follows:

1. A fitness sharing based on niching is utilized to construct an optimization objective to maintain population diversity instead of using niching methods like crowding and species. In this way, TriDEMO is free from the sensitivity of niching parameters.
2. In TriDEMO, an MMOP is transformed into a TOP with strong objective confliction, so that the advantage of evolutionary multiobjective optimization can be fully used.
3. Mathematical proofs are presented to demonstrate that all globally optima of an MMOP are converted into nondominated optimal solutions of the transformed TOP, which provides the feasibility of TriDEMO.
4. We propose a new solution comparison criterion for the multiobjective optimization component as well as an adaptive ranking strategy for DE. As a result, the proposed TriDEMO can obtain the multiple optima with high solution quality.

The effectiveness and efficiency of TriDEMO has been evaluated on 44 benchmark multimodal functions in [26,31]. The performance of TriDEMO is compared with nine state-of-the-art multimodal optimization algorithms, and the experimental results show the advantages of TriDEMO. Moreover, the effectiveness of the third objective, the adaptive ranking strategy, the modified solution comparison criterion, and the parameter settings in TriDEMO are investigated in depth.

The remainder of this paper is organized as follows. Section 2 presents related work on multiobjective optimization techniques for MMOPs, fitness sharing, and DE algorithm. Section 3 develops TriDEMO in detail. Experimental results are reported and analyzed in Section 4. Finally, Section 5 draws the conclusion.

2. Related work

2.1. Multiobjective optimization approaches for MMOPs

Multiobjective optimization techniques [7,40,48], owing to their successes in achieving good balance between population diversity and convergence, have attracted considerable interest in solving various kind of optimization problems [10,35,37]. In order to solve MMOPs by multiobjective optimization approaches, an MMOP at hand should be first transformed into an MOP. Usually, an MOP can be stated as follows:

$$\begin{array}{ll} \text{Minimize} & F(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_m(\mathbf{x})) \\ \text{subeject to} & \mathbf{x} = (x_1, \ldots, x_D) \in \Re \end{array} \tag{1}$$

where D is the number of variables, m is the number of objectives and $\Re$ is the decision space.

Given two vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u}$ Pareto dominates $\mathbf{v}$, if $f_i(u) \le f_i(v)$ for all $i \in \{1, \ldots, m\}$ and $F(\mathbf{u}) \neq F(\mathbf{v})$. Moreover, vector $\mathbf{t}$ is considered as a nondominated solution, if there is no $\mathbf{x} \in \Re$ such that $\mathbf{x}$ Pareto dominates $\mathbf{t}$. In an MOP, the optimization objectives are mutually conflicting, and thus a solution which minimizes all objectives simultaneously does not exist. Hence, there is a set of nondominated solutions for an MOP. The set of all nondominated solutions, is called the Pareto Set, denoted as PS. Vector $F(\mathbf{u})$ is called the Pareto objective vector, if $\mathbf{u}$ is a nondominated solution. The set of all the Pareto objective vectors, $\{F(\mathbf{x})|\mathbf{x} \in PS\}$, is called the Pareto Front.