Exact regulation for disturbed nonlinear systems with unfixe output and uncontrollable/unobservable linearisations

Chuanlin Zhang[1], Pengfei Song[1], Jun Yang[2], Xitong Niu[1]

***Abstract*—The problem of exact regulation control for a class of disturbed higher-order nonlinear systems with unfixe single output and uncontrollable/unobservable linearisations is investigated in this paper. A general composite controller is proposed by integrating the homogeneous domination technique with an observation process of the unmatched disturbances. Inheriting a recursive design procedure and a delicate way of handling the non-vanishing nonlinearities, it is shown that unmatched disturbances in every channel can be compensated in a series of composite virtual controllers design while any state assigned as the system output can be regulated to exact tracking its desired value. A rigorous Lyapunov function based stability analysis and numerical simulations assure the effectiveness of the proposed strategy.**

I. Introduction

Precise regulation problems of the system output have always been the center issue in the development of control theory. In the latest decades, control synthesis for various uncertain nonlinear systems with the advances of novel nonlinear control strategies has been well considered in the literature. Most notably, [1] investigates a homogeneous observer design to render a smooth output feedback control method, [2] develops a generalized homogeneous domination control method which has shown its impact on handling different higher-order nonlinear systems. [3] studies the sampled-data output feedback control problem via a high-gain observer and in [4], the finite-tim control problem for a class of controllable systems is investigated. However, based on a common fact that mismatched disturbances which may enter the system through different channels from those control inputs are frequently encountered in practical systems, advanced control techniques with strong disturbances rejection abilities for the general system (1) in the presence of nonvanishing disturbances should be imperatively investigated.

This work is supported in part by National Natural Science Foundation of China (Nos. 61503236 and 61573099).

1. College of Automation Engineering, Shanghai University of Electric Power, Shanghai, China, 200090. (E-mail: clzhang@shiep.edu.cn)

2. Key Laboratory of Measurement and Control of Complex Systems of Engineering, Ministry of Education, Southeast University, Nanjing, China, 210096.

While various robust control methods have already shown their impacts on practical control implementations, such as H_∞ control [5], sliding mode control [6] and nonlinear output regulation method [7] etc., an alternative style called active anti-disturbance control approach which adopts a disturbance estimation/compensation procedure has aroused researchers' interests for decades [8]. For a wide class of nonlinear systems consisting of linear nominal dynamics, abundant existing results in the literature can be found. For instance, [9] has employed a higher-order sliding mode method to design controller for nonlinear systems with unmatched disturbances. And then it is reported in [10] that a nonsingular terminal sliding mode control method is developed for a class of systems with unmatched disturbances. Deriving from the merits of the backstepping approach, a disturbance observer based backstepping method is studied in [11], in which the unmatched disturbance are compensated by the virtual controllers in a recursive way. [12] studies the finite-tim disturbance rejection problem by integrating a higher-order sliding mode disturbance observer with a finite-tim recursive design method. A recent result [13] presents a novel homogeneous handling strategy for a class of disturbed chain of power integrators. One common way in those mentioned active anti-disturbance methods is to adopt a lumped disturbance consisting the higher-order nonlinearities and other disturbance items with a disturbance observation step but it is clear that their nominal control performance will be sacrifice [8]. Moreover, for nonlinear systems with higher-order nominal dynamics with the presence of unmatched disturbances, the fact that their Jacobian linearisations are uncontrollable/unobservable will clearly lead to an un-applicability of the existing methods.

In a more practical sense, different states are always assigned as the system outputs but meanwhile much complexity will be added into the controller design. Another obstacle in the design is the nonvanishing feature of the involved nonlinearities caused by non-zero unmatched disturbances. In this paper, aiming to fin an active disturbance attenuation control law to reject the adverse effects of the disturbances in the output channel with possibilities of any state, a new composite control

978-1-5090-4657-7/17/$31.00 ©2017 IEEE

method is developed with a delicate handling procedure of the involved nonlinearities and disturbances. Inspired by the existing homogeneous design schemes with its facility in handling nonlinear systems, we integrate the homogeneous domination feedback control with a nonlinear disturbance observation method to yield a homogeneous composite control framework for a wider class of nonlinear systems whose nominal dynamics are no longer required to be linear. Inheriting a Lyapunov function based recursive design procedure in which the unmatched disturbances are estimated and then compensated in every step's virtual controller design, rigorous stability analysis is also provided to show its theoretical justification Moreover, with the given fl xibility of tuning the homogenous degree, the design method could be extended to handle a wider class of nonlinear systems with different nonlinear growth rates. Numerical comparison results are shown to demonstrate the improved control performance of the proposed method.

II. Preliminaries

A. Definition and Notations

The following definition and notations are provided for briefness of expressions.

- The symbols $\mathbb{R}^{+}_{odd}$ and $\mathbb{R}^{\geq 1}_{odd}$ denote the set of ratios of two positive odd integers and set of the ratios that greater than 1, respectively. For integers j and i satisfying $0 \leq j \leq i$, denote $\mathbb{N}_{j:i} := \{j, j+1, \cdots, i\}$. The symbol C^i denotes the set of all differentiable functions whose firs $i-$th time derivatives are continuous. The symbol $\mathcal{L}_\infty$ represents the set of all signals whose infinity-norm are bounded.
- *Weighted Homogeneity [2]:* For a fi ed choice of coordinates $x = (x_1, \cdots, x_n) \in \mathbb{R}^n$, and positive real numbers $(r_1, r_2, \cdots, r_n) \triangleq r$, a one-parameter family of dilation is a map $\prod^r : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$, define by $\prod^r_\epsilon x = (\epsilon^{r_1} x_1, \cdots, \epsilon^{r_n} x_n)$. For a given dilation $\prod^r$ and a real number k, a continuous function $V : \mathbb{R}^n \to \mathbb{R}$ is called $\prod^r$−homogeneous of degree k, denoted by $V \in \mathbb{H}_k$ if $V \circ \prod^r_\epsilon = \epsilon^k V$. A continuous vector fiel $f(x) = \sum f_j(x)(\frac{\partial}{\partial x_j})$ is $\prod^r$− homogeneous of degree τ, if $f_j \in \mathbb{H}_{\tau + r_j}$, $j \in \mathbb{N}_{1:n}$.
- In this paper, we specialize r with $r_1 = 1$, $r_{i+1} p_i \in \mathbb{R}^{+}_{odd} = r_i + \tau > 0$, $i \in \mathbb{N}_{1:n-1}$. The norm $\|\zeta\|_{\prod}$ for a vector $\zeta = [\zeta_1, \cdots, \zeta_n]^T$ is define by $\|\zeta\|_{\prod} = \left(\sum_{j=1}^n \left|\zeta_j^{\frac{a}{r_j+\tau}}\right|^2\right)^{1/2}$. $\Delta^{p_i}(\varsigma) = -\varsigma^{\frac{1}{p_i}} : \mathbb{R} \to \mathbb{R}$ denotes a continuous function.

B. Useful Lemmas

Some useful lemmas are stated as follows for convenience of the readers [14].

Lemma 2.1: Let p be a ratio of positive odd integers. If $0 < p \leq 1$, then $|x^p - y^p| \leq 2^{1-p}|x-y|^p$, $(|x| + |y|)^p \leq |x|^p + |y|^p$. Moreover, if $p \geq 1$, we have $|x+y|^p \leq 2^{p-1}|x^p + y^p|$, $|x^p - y^p| \leq c|x-y|(|x-y|^{p-1} + |y|^{p-1})$.

Lemma 2.2: Let c, d be positive constants. Given any positive smooth function $\gamma(x, y)$, then the following inequality hold $|x|^c|y|^d \leq \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}(x,y)|y|^{c+d}$.

III. Problem Formulation

In this paper, we study the active disturbance attenuation control problem for the following disturbed nonlinear system

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}^{p_i}(t) + f_i(t, \bar{x}_i(t)) + d_i(t),\ i \in \mathbb{N}_{1:n-1}, \\ \dot{x}_n(t) &= u^{p_n}(t) + f_n(t, x(t)) + d_n(t), \\ y(t) &= x_m(t),\ 1 \leq m \leq n, \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T$, $x = \bar{x}_n$, y, y_r and u are system partial state vector, full state vector, output, output constant reference signal and control input, respectively. $d = [d_1, \cdots, d_n]^T$ is an unmatched disturbance vector, $p_i \in \mathbb{R}^{\geq 1}_{odd}$, $i \in \mathbb{N}_{1:n}$ with $p_n = 1$ and $f_i(t, \bar{x}_i(t))$, $i \in \mathbb{N}_{1:n}$ is a known C^1 nonlinear function. Assume the system satisfie the following assumptions:

Assumption 3.1: There exist constants $\tau \geq 0$ and $\gamma_i > 0$ such that $|f_i(t, \bar{x}_i) - f_i(t, \hat{\bar{x}}_i)| \leq \gamma_i \sum_{j=1}^i |x_j - \hat{x}_j|^{\frac{r_i+\tau}{r_j}}$.

Assumption 3.2: The unmatched disturbance and its firs order derivative satisfy $d(t) \in \mathcal{L}_\infty$, $\dot{d}(t) \in \mathcal{L}_\infty$. Moreover, there exists a finite-tim T^*, such that $\dot{d}(t) = 0$, $\forall t \geq T^*$.

Remark 3.1: With a given fl xibility of tuning the homogeneous degree τ, this assumption has covered a wide class of inherent nonlinear system with higher-order nonlinearities and unmatched disturbances. For instance, polynomial functions with the form of $\sum_{j=1}^i x_i^{\rho_j}$ with $\rho_j \geq 1$ being a real number can be easily satisfied Moreover, there are some special smooth functions, such as $\sin(x)$, $\cos(x)$, $\ln(1 + x^2)$ and $\arctan(x)$ can also be located within this assumed property. With a special case when $\hat{\bar{x}}_i = 0$, Assumption 3.1 reduces to a homogeneous growth condition in [2].

Remark 3.2: Note that Assumption 3.1 includes a general nonvanishing condition, rather than the commonly used vanishing condition (i.e., $f_i(0) = 0$) utilized in [2]. It poses a more challenging problem to regulate the states to their desired equilibrium. For instance, consider a 2-D system of the following form

$$\dot{x}_1 = x_2 + \cos(x_1) + d(t),\ \dot{x}_2 = u,\ y = x_2. \tag{2}$$

Since $\cos(0) \neq 0$ and the control objective is to regulate $y \to 0$ under the unmatched disturbance $d(t)$. Existing