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# Dissipativity-based asynchronous filtering for periodic Markov jump systems

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ABSTRACT

This paper is devoted to developing a periodic and asynchronous filter for periodic Markov jump systems. A periodic and asynchronous filter means that the filter parameters are *s*-periodic for a given operating mode, and its operating modes are unnecessary to be identical but form a periodic hidden Markov model together with the modes of the plant. A sufficient condition is proposed to guarantee that the filtering error system is exponentially mean square stable and strictly $(\mathscr{Q}, \mathscr{S}, \mathscr{R})$-dissipative. To realize this filter, a slack matrix is introduced to deal with the nonlinearity and Finsler's lemma is applied such that the decision variables are reduced. Finally, a numerical simulation provides verification for the effectiveness of our designed filter.



## 1. Introduction

Markov jump systems (MJSs) have enjoyed a boom during the past decades thanks to the mighty modeling capability of MJSs in many areas (see e.g., [1,8,21]). As for the vertical take-off landing helicopter, Markov process is applied to describe the airspeed variations [8]. The solar thermal receiver can be described by a set of system modes due to sudden changes of insolation level [21]. It is also efficient to apply MJS to model communication constraints in networked control systems [1]. Substantial efforts have been devoted to the study of MJSs, [5,10,12,13,15,17–20,22,25,28,32–34] are some of the recent works, and in which, different issues have been addressed, systems with various attributes have been considered and different schemes have been applied. For example, [15,28] have been concerned with sliding mode control, [20,32,33] have studied the problem of filtering/estimation, and balanced truncation has been discussed in [13]. Piecewise homogeneous Markov chain, semi-Markov chain and nonhomogeneous chain with a polytopic transition probability matrix has been considered in [32–34], respectively.

In practice, the parameters of the system always exhibit time-variant properties due to the influence of many factors, e.g. the variation of external environment. As a special case of time-variant systems, periodic system is universal in reality, e.g., natural sciences, economics and finance [3]. For example, economic data always includes a seasonal term. In satellite attitude

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control, a time history of geomagnetic field experienced by the satellite along a polar orbit exhibits strong periodicity[23]. The early stage of study on periodic systems can data back to 1880s and later this special class of systems have been vastly analyzed and synthesized by extending the framework of LTI systems, the existing results can be found in [3]. Recently, the periodicity has been introduced into MJSs. Observability and detectability of periodic systems with nonhomogeneous Markov jump parameter have been analyzed in [11]. Aberkane and Dragan [1] has established a necessary and sufficient condition to guarantee that the discrete-time periodic MJSs are exponentially mean square stable and satisfy certain $H_\infty$ performance under an assumption that the system is weak controllable.

It is noted that, in many existing results on MJSs, a common assumption that the plant and filter or controller run synchronously with each other is adopted, see e.g. [10,13,15,17–19,25,28,32–34]. An important implication by synchronization is that all the modes of the plant are fully accessible to the filter or controller, which is rather rigid in reality. For instance, there is always information loss in networked systems as a result of the imperfection of transmission channel. On the other hand, it is always costly to estimate the modes of the plant in real time. As a result, there is growing concern about asynchronous filtering or control issues and different schemes are proposed to describe the asynchronization phenomenon. In [16,29,30], there exist time delays between the mode of the plant and that of the controller, which leads to asynchronization. The asynchronization in [26,31,35] has been characterized by a piecewise homogeneous Markov chain, i.e., the Markov chain regulating the variations of the filter is piecewise homogeneous. What is special in [7] is that the time delay is mode-dependent and asynchronous with the plant. Wu et al. [27] has modeled the asynchronization phenomenon as hidden Markov model which covers synchronous and mode-independent cases. Hidden Markov model has been used to deal with anomaly network intrusion detection in [6], but it is the first time in [27] to use this model to address asynchronization. However, dissipativity-based asynchronous filtering for periodic MJSs remains a problem to be resolved, which greatly stimulates our interest.

The notion of dissipativity has been studied for along time, which was initiated by Willems [24]. Generally speaking, dissipativity means the energy storage is no more than energy supply, which implies that there exists energy dissipation in the systems. Usually, the storage function provides natural candidate for Lyapunov function. Practical examples of dissipative systems include electrical networks and viscoelastic systems. Taking the electrical networks as an example, part of the electrical energy is consumed by the resistors in the form of heat. Furthermore, the notion of dissipativity has been generalized to left-continuous dynamical systems which covers switching systems in [9]. Consequently, it is desirable to use dissipativity to analyze MJSs.

The filtering problem to be settled in this paper features periodicity of the considered MJS and the asynchronization between the MJS and the designed filter. To be more specific, the main contributions of our work can be summarized as follows: 1) Under the framework of periodic hidden Markov model which is introduced to depict the asynchronization between the MJS and filter, a sufficient condition is derived such that the exponential mean square stability and strict $(\mathscr{Q},\mathscr{S},\mathscr{R})$-dissipativity of the filtering error system is guaranteed. 2) The parameters of the asynchronous filter is further parameterized through LMI method. It is worth mentioning that the efficiency of the proposed design method is improved by applying Finsler's lemma.

## 2. Preliminaries

A class of discrete-time MJSs, defined in a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, is considered in this paper, which is described as follows:

$$S_0: \begin{cases} x(k+1) = A(k,\varepsilon_k)x(k) + B(k,\varepsilon_k)w(k) \\ y(k) = C_1(k,\varepsilon_k)x(k) + D_1(k,\varepsilon_k)w(k) \\ z(k) = C_2(k,\varepsilon_k)x(k) + D_2(k,\varepsilon_k)w(k) \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state vector, $y(k) \in \mathbb{R}^{n_y}$ is the measurable output, $z(k) \in \mathbb{R}^{n_z}$ is the signal to be estimated, and $w(k) \in \mathbb{R}^{n_w}$ which belongs to $l_2[0,\infty)$ is the external perturbance input. The process $\{\varepsilon_k, k \geq 0\}$ is characterized by a discrete-time Markov chain which is confined to a finite set $\mathscr{N}_1 = \{1, 2, \cdots, n_1\}$ and its pre-known transition probability matrix $\Lambda(k) = \{\lambda_{ij}(k)\}$ is given by

$$\Pr\{\varepsilon_{k+1} = j | \varepsilon_k = i\} = \lambda_{ij}(k) \tag{2}$$

which is subject to $\lambda_{ij}(k) \geq 0$ and $\sum_{j=1}^{n_1} \lambda_{ij}(k) = 1$ for $\forall i, j \in \mathscr{N}_1$, and $k \geq 0$. Since the transition probability matrix $\Lambda(k)$ depends on time instant $k$, this Markov process is nonhomogeneous, which covers the homogeneous case if let $\Lambda(k) = \Lambda$.

In this paper, we turn to the problem of filter design for the aforementioned system. Here, a full-order filter of the following form is adopted:

$$S_f: \begin{cases} x_f(k+1) = A_f(k,\eta_k)x_f(k) + B_f(k,\eta_k)y(k) \\ z_f(k) = C_f(k,\eta_k)x_f(k) + D_f(k,\eta_k)y(k) \end{cases} \tag{3}$$

where $x_f(k)$ is the state vector of the filter, and $z_f(k)$ is the estimation of $z(k)$. $A_f(k,\eta_k)$, $B_f(k,\eta_k)$, $C_f(k,\eta_k)$ and $D_f(k,\eta_k)$ are time-varying filter parameters to be determined, which depend on the parameter $\eta_k$ taking values in the finite set $\mathscr{N}_2 = \{1, 2, \cdots, n_2\}$ with a preset conditional probability matrix $\Sigma = \{\sigma_{im}\}$, the conditional probability is defined by

$$\Pr\{\eta_k = m | \varepsilon_k = i\} = \sigma_{im} \tag{4}$$