Information Sciences 418–419 (2017) 317–345

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

A modified PSO algorithm for linear optimization problem subject to the generalized fuzzy relational inequalities with fuzzy constraints (FRI-FC)

Amin Ghodousian[a,*], Maryam Raeisian Parvari[b]

[a] Department of Engineering Science, College of Engineering, University of Tehran, Tehran 11365-4563, Iran
[b] Department of Algorithms and Computation, University of Tehran, Tehran, Iran

ARTICLE INFO

Article history:
Received 21 November 2016
Revised 25 June 2017
Accepted 26 July 2017
Available online 8 August 2017

Keywords:
Fuzzy relational inequalities
Linear optimization
t-norm
PSO algorithm
Fuzzy constraints

ABSTRACT

In this paper, optimization of a linear objective function subject to a generalized fuzzy relational inequalities is investigated in which an arbitrary continuous t-norm is considered as fuzzy composition, and fuzzy inequality replaces ordinary inequality in the constraints. Unlike most optimization algorithms, in fuzzy relational inequalities with fuzzy constraints (FRI-FC) we find a near-feasible solution having a better objective value than those resulting from the resolution of the similar problems with crisp (ordinary) inequality constraints. Such solutions are called super-optima in this paper. Subsequently, an algorithm is proposed to find a super-optimum with pre-specified desirable infeasibility. For this purpose, we firstly study some structural properties of the FRI-FC problem and present a new formulization that is independent of t-norms used in the constraints of the problem. This new formulation converts the primary problem into an equivalent problem with simple constraints without considering any penalty parameters. However, it is proved that the transformed equivalent problem enables our algorithm to distinguish unfavorable points and generate a sequence of solutions converging to a super-optimum under a certain sufficient condition. Finally, a modified PSO is presented in which the ability of PSO to solve unconstrained problems with continuous domain, the structure of the transformed problem and some fuzzy structural modifications are combined and form an efficient algorithm to solve the generalized FRI-FC problems. The modified PSO algorithm has been applied to the generalized FRI-FC problem defined with ten well-known continuous t-norms. Additionally, an idea of the FRI-FC problems as outer approximators for FRI problems with ordinary inequalities is also investigated.

1. Introduction

The theory of fuzzy relational equations (FRE) as a generalized version of Boolean relation equations was firstly proposed by Sanchez and applied in problems of the medical diagnosis [1]. Nowadays, it is well known that many issues associated with a body knowledge can be treated as FRE problems [2]. In addition to such applications, FRE theory has been applied in many fields including fuzzy control, discrete dynamic systems, prediction of fuzzy systems, fuzzy decision making, fuzzy

* Corresponding author.
E-mail address: a.ghodousian@ut.ac.ir (A. Ghodousian).

http://dx.doi.org/10.1016/j.ins.2017.07.032
0020-0255/

pattern recognition, fuzzy clustering, image compression and reconstruction, fuzzy information retrieval, and so on. Generally, when inference rules and their consequences are known, the problem of determining antecedents is reduced to solving an FRE [3]. We refer the reader to [4] in which the authors provided a good overview of FRE and classified basic FREs by investigating the relationship among operators used in the definition of fuzzy relational equations.

The solvability determination and the finding of solutions set are the primary (and the most fundamental) subject concerning FRE problems. Di Nola et al. proved that the solution set of FRE (if it is nonempty) defined by continuous max-t-norm composition is often a non-convex set that is completely determined by one maximum solution and a finite number of minimal solutions [5]. This non-convexity property is one of two bottlenecks making major contribution to the increase in complexity in problems that are related to FRE, especially in the optimization problems subjected to a system of fuzzy relations. The other bottleneck is concerned with detecting the minimal solutions for FREs. Chen and Wang [6] presented an algorithm for obtaining the logical representation of all minimal solutions and deduced that a polynomial-time algorithm to find all minimal solutions of FRE (with max–min composition) may not exist. Also, Markovskii showed that solving max-product FRE is closely related to the covering problem which is an NP-hard problem [7]. In fact, the same result holds true for more general t-norms instead of the minimum and product operators [6,8–10]. Over the last decades, the solvability of FRE defined with different max-t compositions has been investigated by many researchers [11–19]. Moreover, some researchers introduced and improved theoretical aspects and applications of fuzzy relational inequalities (FRI) [20–25]. Li and Yang [24] studied an FRI with addition-min composition and presented an algorithm to search for minimal solutions. They applied FRI to meet a data transmission mechanism in a BitTorrent-like Peer-to-Peer file sharing systems. In [20], the authors focused on the algebraic structure of two fuzzy relational inequalities $A\varphi x \le b^1$ and $D\varphi x \ge b^2$, and studied a mixed fuzzy system formed by the two preceding FRIs, where φ is an operator with (closed) convex solutions. Generally, if φ is an operator with closed convex solutions, the solutions set of $D\varphi x \ge b^2$ is determined by a finite number of maximal solutions as well as the same number of minimal ones. In particular, if φ is a continuous non-decreasing function (specially, a continuous t-norm), all maximal solutions overlap one another [20]. Guo et al. [22] investigated a kind of FRI problems and the relationship between minimal solutions and FRI paths. They also introduced some rules for reducing the problems and presented an algorithm for solving optimization problems with FRI constraints.

The problem of optimization subject to FRE and FRI is one of the most interesting and on-going research topics among the problems related to FRE and FRI theory [20–23,25–38]. Fang and Li [39] converted a linear optimization problem subjected to FRE constraints with max–min operation into an integer programming problem and solved it by branch-and-bound method using jump-tracking technique. In [40], an application of optimizing the linear objective with max–min composition was employed for the streaming media provider seeking a minimum cost while fulfilling the requirements assumed by a three-tier framework. Wu et al. [41] improved the method used by Fang and Li, via decreasing the search domain and presented a simplification process by three rules resulted from a necessary condition. Chang and Shieh [26] presented new theoretical results concerning the linear optimization problem constrained by fuzzy max–min relation equations. They improved an upper bound on the optimal objective value, some rules for simplifying the problem and proposed a rule for reducing the solution tree. The topic of the linear optimization problem was also investigated with max-product operation [28,42,43]. Loetamonphong and Fang defined two sub-problems by separating negative and non-negative coefficients in the objective function and then obtained the optimal solution by combining those of the two sub-problems [43]. The maximum solution of FRE is the optimum of the sub-problem having negative coefficients. Another sub-problem was converted into a binary programming problem and solved by branch and bound method. Also, in [42] and [28], some necessary conditions of the feasibility and simplification techniques were presented for solving FRE with max-product composition. Moreover, some generalizations of the linear optimization with respect to FRE have been studied with the replacement of max–min and max-product compositions with different fuzzy compositions such as max-average composition [32,38], max-star composition [30,33] and max-t-norm composition [31,34,37]. For example, Li and Fang [34] solved the linear optimization problem subjected to a system of sup-t equations by reducing it to a 0–1 integer optimization problem. In [31] a method was presented for solving linear optimization problems with the max-Archimedean t-norm fuzzy relation equation constraint. In [37], the authors solved the same problem whit continuous Archimedean t-norm and used the covering problem rather than the branch-and-bound methods for obtaining some optimal variables.

Recently, many interesting generalizations of the linear programming subject to a system of fuzzy relations have been introduced and developed based on composite operations used in FRE, fuzzy relations used in the definition of the constraints, some developments on the objective function of the problems and other ideas [35,44–48]. For example, Wu et al. [48] represented an efficient method to optimize a linear fractional programming problem under FRE with max-Archimedean t-norm composition. Dempe and Ruziyeva [44] generalized the fuzzy linear optimization problem by considering fuzzy coefficients. Dubey et al. studied linear programming problems involving interval uncertainty modeled using intuitionistic fuzzy set [45]. The linear optimization of bipolar FRE was studied by some researchers where FRE was defined with max–min composition [46] and max-Lukasiewicz composition [35,47]. For example, in [47] the authors introduced a linear optimization problem subjected to a system of bipolar FRE defined as $X(A^+, A^-, b) = \{x \in [0, 1]^m : x \circ A^+ \vee \tilde{x} \circ A^- = b\}$ where $\tilde{x}_i = 1 - x_i$ for each component of $\tilde{x} = (\tilde{x}_i)_{1\times m}$ and the notations "$\vee$" and "$\circ$" denote max operation and the max-Lukasiewicz composition, respectively. They translated the problem into a 0–1 integer linear programming problem which is then solved using well-developed techniques. In [35], the foregoing bipolar linear optimization problem was solved by an analytical method based on the resolution and some structural properties of the feasible region (using a necessary condition for characterizing an optimal solution and a simplification process for reducing the problem).