Matrix Completion for Downward-Looking 3-D SAR Imaging With a Random Sparse Linear Array

Siqian Zhang, *Member, IEEE*, Ganggang Dong, and Gangyao Kuang, *Senior Member, IEEE*

***Abstract*— Downward-looking linear array 3-D synthetic aperture radar (SAR) has attracted increasing attention in the field of radar imaging. As widely reported, the volume of data can be significantly reduced by a random sparse linear array. However, the 2-D under-sampled azimuth-cross-track data brought by the sparse linear array will produce high-level side-lobes, as well as the aliasing and the false-alarm targets. To deal with those problems, this paper introduces a recently developed theory, matrix completion (MC). The new theory could recover a matrix with a small subset of known elements of the matrix. It is founded on the assumption that the matrix is essentially low rank. For downward-looking 3-D SAR with a random sparse linear array, the received 3-D data can be treated as a series of uncorrelated 2-D matrices by the separated channel process. First, range compression can be realized by means of pulse compression. Then, the sets of the 2-D under-sampled azimuth-cross-track matrix can be completed into a full-sampled one via MC trick. The resulting 3-D images can be focused by synthetic aperture technique along the azimuth direction and beamforming operation along the cross-track direction, with the recovered full-sampled matrix. The proposed algorithm achieves high resolution and low-level side-lobes with the acceptable computational cost and memory consumption. It is verified by several numerical simulations and multiple comparative studies on real data. The experimental results clearly demonstrate the imaging performance across different under-sampling rates and signal-noise rates.**

***Index Terms*— 3-D imaging, data recovery, matrix completion (MC), synthetic aperture radar (SAR), under-sampled data.**

I. Introduction

THREE-dimensional synthetic aperture radar (3-D SAR) imaging has been one of the research hotspots in recent years, thanks to the rapid development of new high-resolution radar sensors. Generally, high-resolution 3-D SAR imaging techniques can be categorized into three parts: interferometric SAR (InSAR), curvilinear SAR (CSAR), and linear array SAR. InSAR exploits two or more antennas looking at the scene with slightly different view angles to estimate the 3-D altimetric profile of the imaged scene [1]. However, the generation of the digital elevation model depends on precise baseline estimation and the scatterers distribution along the height direction cannot be obtained [1]–[3]. CSAR [4]–[7], employing the curvilinear aperture via the maneuver trajectory on the azimuth-height plane, has the large aperture both in the azimuth and height direction to obtain a 3-D imaging capability. It is difficult to control the precise flight trajectory. Moreover, these SAR systems work in the side-looking mode, which results in shading and layover effects. To overcome these restrictions, a new method, downward looking imaging radar, is presented by Gierull [8]. The new technique combines the concepts of real and synthetic apertures to generate 2-D images with a single frequency transmitted signal. Nouvel *et al.* [9]–[11] in ONERA developed the concept of downward looking 3-D SAR. Weib and Ender [12] and Klare *et al.* [13], [14] designed an airborne radar for 3-D imaging and nadir observation (ARTINO). ONERA [9] and ARTINO [12], as the representative systems of downward-looking linear array 3-D SAR (DLLA 3-D SAR), can generate a high-resolution 3-D imagery of a directly overflown scene.

DLLA 3-D SAR acquires a satisfactory height resolution by emitting signals with a larger bandwidth, whereas a finer 2-D resolution on the azimuth and cross-track plane is achieved by synthetic aperture technique and beamforming operation, respectively [15]–[18]. Due to its advanced performances, a great many works have done on DLLA 3-D SAR imaging, e.g., 3-D range migration algorithm [19], 3-D chirp scaling algorithm [20], 3-D polar format algorithm [21], and so on. These techniques require an uniform real aperture in the cross-track direction, which can be performed by using a linear array with several equally spaced antenna elements. However, the demand for a high resolution leads to the high sampling rate based on the Nyquist theorem, which poses the challenges for 3-D raw data storage and transmission. In order to alleviate the 3-D data storage and transmission burden, a sparse linear array is applied for DLLA 3-D SAR to acquire a 2-D under-sampled azimuth-cross-track raw data. Employing a small number of samples enables power saving during the communication phase between the receive antennas and the fusion center, which is critical for enabling applications of DLLA 3-D SAR in power limited scenarios, e.g., when the antennas are placed on battery operated nodes [22]. In addition, the sparse linear is unavoidable under some special structure limits of the wingspan, e.g., engines hung under the wingspan. Obviously, DLLA 3-D SAR with a random sparse linear array is more tally with the actual situation.

The tradeoff in using sparse data is their unpredictable sidelobe behavior. Since the under-sampling rate (USR) no longer satisfies the Nyquist sampling rate, the performance of 3-D images is degraded drastically. The targets even cannot

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The authors are with the College of Electronic Science and Engineering, National University of Defense Technology, Changsha 410073, China (e-mail: zhangsiqian@nudt.edu.cn).

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be focused by these conventional 3-D imaging algorithms. Furthermore, the techniques based on spectral estimation have been proposed and applied for 3-D imaging to achieve a higher resolution [23]–[25]. Nonetheless, because the received data in DLLA 3-D SAR with a random sparse linear array is sparse on the azimuth-cross-track plane, the response of the system matched filter has severe high side-lobes and the acceptable images cannot be generated via nonparametric methods from the received data [26]. All methods mentioned above and working on DLLA 3-D SAR with a random sparse linear array are not capable of providing 3-D images with a high resolution. Although compressive sensing (CS) [27]–[29] has been applied in 3-D SAR for data acquisition and signal recovery based on fewer signal samples, it is unavoidable to construct a measurement matrix. As a result, the quality of the 3-D image may be seriously affected by the accuracy of its measurement matrix [30].

Matrix completion (MC) [31]–[33] is a technique for realizing the reconstruction of a low rank data matrix by using a set of limited observations of its entries. For under-sampled data, if the low-rank property of a reshaped data matrix is satisfied, the unobserved data can be recovered from the known samples by nuclear norm minimization [22], [32], [34]. The effectiveness and advantages of MC for the conventional SAR imaging were presented in [30], whereas MC method is only applied to deal with 1-D under-sampled data in the azimuth direction. For DLLA 3-D SAR with a random sparse linear array, the 2-D matrix signals on the azimuth-cross-track plane are sparse, due to the under-sampling along the cross-track direction by the sparse linear array. Hence, the unobserved samples of these 2-D matrix signals need to be recovered from the known samples by MC theory.

In this paper, a novel imaging algorithm based on MC is proposed for DLLA 3-D SAR with a random sparse linear array. Although under-sampled data can effectively relieve the pressure of data storage and transmission, the invalid of the Nyquist sampling rate causes the poor performance of the 3-D images with the conventional 3-D imaging algorithms based on the matched filtering. To solve this problem, the MC method can be used to complete the under-sampled 2-D matrix signals on the azimuth-cross-track plane brought by the sparse linear array into the full-sampled ones. Based on the recovered full-sampled data, the 3-D images can be focused with a high resolution and low side-lobes. Our contributions are therefore fourfold.

1) We derive the 3-D signal model of DLLA 3-D SAR with a random sparse linear array.
2) We prove that the 2-D sparse matrix signals on the azimuth-cross-track plane is essentially of low rank.
3) We recover the missing entries from the 2-D sparse matrix signal based on MC method to construct a full-sampled data.
4) To demonstrate the advantage and validity of the proposed algorithm, we do extensive experiments on simulated and real data under different signal-noise rates (SNRs) and USRs.

This paper is organized as follows. Section II establishes the 3-D signal model of DLLA 3-D SAR with a random sparse linear array. In Section III, we present a 3-D imaging algorithm via MC for DLLA 3-D SAR with a random sparse linear array and discuss the low rank property of 2-D sparse matrix signal. Section IV investigates that the validity and the advantage of the proposed algorithm have been verified by the 3-D scene simulation and experiments on real data. The performance of the proposed algorithm is demonstrated via different USRs and SNRs.

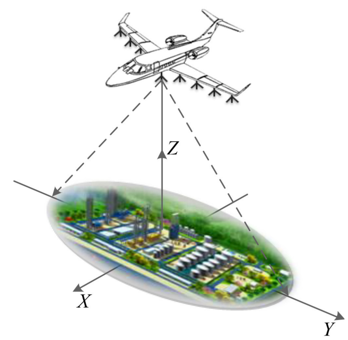

Fig. 1. Imaging geometric model of DLLA 3-D SAR with a random sparse linear array.

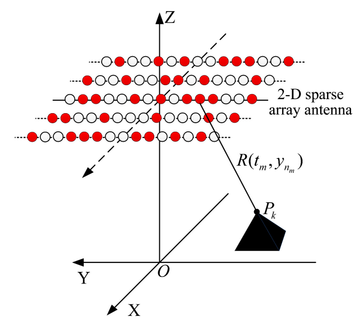

Fig. 2. Geometric model of 2-D sparse array antenna on the azimuth-cross-track plane. Red solid dots: active antenna elements. Hollow dots: inactive antenna elements.

II. Signal Model of DLLA 3-D SAR With a Random Sparse Linear Array

Let us consider the imaging geometry of DLLA 3-D SAR with a random sparse linear array shown in Fig. 1. The plane is supposed to fly at the altitude H along the flight path of the platform (X-axis) with the velocity v. Axes Y and Z are the cross-track and range (height) directions, orthogonal to each other, and orthogonal to X. The orbits and sparse linear array construct a 2-D sparse array antenna on the azimuth-cross-track plane (see Fig. 2). The linear array is composed of N antenna elements, which are equally spaced with distance d. The length of the linear array is $L_y = (N-1)d$. At each slow time t_m, the antenna elements are randomly activated to form a sparse linear array y_{n_m} $(1 \le n_m \le N_m)$, where N_m is the number of active antenna elements at slow time t_m. The positions of the array elements are described by

$$P_{t_m} = \{P_{mn_m} | P_{mn_m} = (x_m, y_{n_m}, H)\}. \quad (1)$$

The SAR illuminates the area of interest from N_m active antenna elements and receives the signal scattered by the scene. The antenna array operates in the monostatic mode. Assuming that the kth point scatterer is located at $P_k = (x_k, y_k, z_k)$, the instantaneous distance R can be expressed as

$$\begin{aligned} R(t_m, y_{n_m}) &= \sqrt{(x_m - x_k)^2 + (y_{n_m} - y_k)^2 + (H - z_k)^2} \\ &= \sqrt{R_0^2 + x_m^2 - 2x_m x_k + y_{n_m}^2 - 2y_{n_m} y_k} \quad (2) \end{aligned}$$