# Hub-and-spoke network design problem under uncertainty considering financial and service issues: A two-phase approach

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## ABSTRACT

This paper proposes a bi-objective hub-and-spoke (H&S) network design problem with type-2 (T2) fuzzy transportation cost and travel time described by parametric secondary possibility distributions, which are obtained using three types of mean value (MV) reduction methods. The considered objectives jointly minimize the generalized expectation of the total transportation costs and the maximum travel time requirement in terms of generalized value-at-risk (VaR). To solve the fuzzy bi-objective H&S network design problem, we develop a two-phase approach, where in the first phase we convert the proposed model into its equivalent parametric mixed-integer programming problems by applying an equivalent transformation method. This is followed by the second phase using a fuzzy linear programming approach implemented with an augmented max-min operator to obtain a non-dominated solution that has an equal satisfactory degree on both objectives. Finally, a case study based on the Civil Aeronautics Board (CAB) data set is conducted to demonstrate the effectiveness of the proposed model and solution approach.

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## 1. Introduction

### 1.1. Background

Hub-and-spoke (H&S) networks have recently received increased attention due to their multiple applications in public transportation, logistics distribution systems, and telecommunications. As a key element of H&S networks, hubs are centralized facilities that serve for consolidating, switching and sorting and allowing for the replacement of direct connections between all nodes with fewer and indirect connections. H&S network design problems are therefore to locate hub facilities and discounted transportation links, allocate origin and destinations nodes (spokes) to hubs, and route flows through the network. From the aspect of operation, the performance of an H&S network can be gauged on different metrics, such as transportation cost, travel time, reliability, flexibility and safety [11]. In this research, the total transportation costs for all origin-destination (O-D) pairs and the maximum travel time requirement between any O-D pair are used as the performance criteria. More specifically, the total transportation costs are related to the benefit of the company, and the maximum travel time requirement is used to evaluate the service quality for customers. It is exactly based on this consideration that this study focuses on the financial and service aspects of H&S networks.

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Designing an H&S network for a company corresponds to a long-term strategic decision, which is typically within an uncertain circumstance occurring when working in a dynamic and chaotic environment. That is, transportation costs, travel times and other parameters cannot always be exactly determined and known in advance. There are several reasons for this, such as lack of transportation information, multiple sources of data, fluctuating nature of parameter values, noise in data, poor statistical analysis, uncertainty in judgment, and so forth. Theoretically, such types of uncertainty can be characterized by type-2 (T2) fuzzy variables by professional judgments or empirical estimates where the membership degree of each point cannot be exactly determined. The three-dimensional nature of a T2 fuzzy variable provides an extra degree of freedom to represent the uncertainty and fuzziness of the real world for application in H&S network design problems. By combining these aforementioned aspects, in this research, we are particularly interested in providing insights into how a company can configure H&S networks to be both efficient (low transportation cost) and effective (high service quality) under uncertainty.

### 1.2. Literature review

The H&S network design problem is conventionally called the hub location-allocation problem, which primarily consists of the hub median problem and the hub center problem. The study of hub location-allocation was formally proposed by O'Kelly [27,28], who provided a quadratic integer programming formulation for the hub median problem. Skorin-Kapov et al. [35] obtained exact solutions to the hub median problem by developing tight linear relaxations of the formulation. Campbell [4] proposed the first formulation for the hub center problem as a quadratic programming model. Kara and Tansel [12] provided several linear formulations for hub center problems. New MILP formulations of the hub location-allocation problem with fewer variables and constraints were developed by Ernst and Krishnamoorthy [9]. For a detailed review of the hub location-allocation problem and its variations, please see Alumur and Kara [1] and Farahani et al. [10].

The purpose of this paper is to study the hub location-allocation problems under uncertainty, which is an active research area in the literature. The significance of uncertainty has motivated some researchers to address hub location problems with random parameters. For example, Marianov and Serra [22] focused on stochasticity at the hub nodes by representing hub airports as M/D/c queues and limiting the number of airplanes that can queue at an airport through chance constraints. Yang [37] presented a two-stage stochastic programming model for air freight hub location and flight route planning under seasonal demand variations. Contreras et al. [8] studied the stochastic uncapacitated hub location problem in which uncertainty is associated with demands and transportation costs. Sim et al. [34] attempted to address hub location-allocation with stochastic time and utilized a chance-constrained formulation to model the minimum service-level requirement. Mohammadi et al. [25] proposed a new stochastic multi-objective multi-mode transportation model for the hub location-allocation problem under uncertainty. Moreover, some new methods have also been developed to model hub location-allocation problems under possibilistic uncertainty. For instance, Taghipourian et al. [36] presented a fuzzy integer linear programming approach for the dynamic virtual hub location problem with the aim of minimizing the transportation cost in a network. Chou [7] proposed a fuzzy multiple criteria decision-making model for evaluating and selecting the container transshipment hub port. Yang et al. [38], 39] presented two classes of hub location-allocation problems with fuzzy travel times based on the credibility criterion. Mohammadi and Moghaddama [26] proposed a bi-objective fuzzy hub location-allocation problem with the choice of a transportation mode over inter-hub links by incorporating a fuzzy M/M/1 queuing system.

In the fuzzy community, it is well known that Zadeh [41] first proposed the concept of a type-2 (T2) fuzzy set as an extension of the ordinary fuzzy set. A T2 fuzzy set is characterized by a fuzzy membership function, where the degree of membership for any element in this set is a fuzzy number in the interval [0, 1]. Since then, T2 fuzzy set theory has been well studied in the literature [5,6,16,23,24,30,33]. Among them, Liang and Mendel [16] proposed the concept of interval T2 fuzzy sets, which can address the operations via interval arithmetics; Mendel and John [23] noted that a T2 fuzzy set represents the uncertainty in terms of a secondary membership function and footprint of uncertainty; Mitchell [24] used the concept of embedded type-1 (T1) fuzzy numbers to provide a method for ranking T2 fuzzy numbers; Chen and Chang [6] proposed a new method for fuzzy rule interpolation for sparse fuzzy rule-based systems with the ratio of fuzziness of interval T2 fuzzy sets; and Qin et al. [33] extended the VIKOR method based on prospect theory for multiple attribute decision making under an interval T2 fuzzy environment. In fuzzy possibility theory [20], a T2 fuzzy variable is a variable-based approach for handling T2 fuzziness, and it is characterized by a T2 possibility distribution function with a three-dimensional structure. The possibility that a T2 fuzzy variable takes its value is a regular fuzzy variable (RFV), which is easier to be determined than a crisp number in practical applications. Therefore, a T2 fuzzy variable is a more advisable tool for characterizing fuzziness than an ordinary fuzzy variable. To reduce uncertainty in the secondary membership function, Karnik and Mendel [13] proposed a defuzzification method with the concept of centroid of a T2 fuzzy set; Liu [19] proposed a centroid-type reduction strategy for a general T2 fuzzy logic system; and Qin et al. [31], 32] developed the critical value (CV) and mean value (MV) reduction methods based on nonlinear fuzzy integrals. Based on the possibility measure, Yang et al. [40] reduced the uncertainty embedded in the secondary possibility distribution of a T2 fuzzy variable by a fuzzy integral, and Bai and Liu [3] developed VaR-based reduction methods for T2 fuzzy variables.

A number of conclusions from the survey of the literature review can be drawn, including the following:

- Most formulations for the H&S network design problem consider transportation costs and travel times separately in the literature.
- The number of research works that address T2 fuzzy uncertainty in the H&S network design problem is fairly small.