Information Sciences 402 (2017) 105–123

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

Acceptability measurement and priority weight elicitation of triangular fuzzy multiplicative preference relations based on geometric consistency and uncertainty indices

Zhou-Jing Wang*, Jian Lin

School of Information, Zhejiang University of Finance & Economics, Hangzhou, Zhejiang 310018, China

ARTICLE INFO

Article history:
Received 20 August 2016
Revised 15 March 2017
Accepted 22 March 2017
Available online 23 March 2017

Keywords:
Multi-criteria decision making
Triangular fuzzy multiplicative preference relation
Consistency
Acceptability
Triangular fuzzy multiplicative weight

ABSTRACT

This paper investigates consistency and uncertainty measurements as well as acceptability checking for triangular fuzzy multiplicative preference relations (TFMPRs). A geometric consistency index is proposed to measure the consistency degree of a TFMPR, and thresholds of acceptable consistency are discussed for TFMPRs. Uncertainty indices are introduced to measure indeterminacy degrees of triangular fuzzy judgments and of TFMPRs and a notion of acceptable TFMPRs are put forward by considering both acceptable consistency and acceptable uncertainty in TFMPRs. We also explore the elicitation of triangular fuzzy weights from acceptable TFMPRs and the aggregation of local triangular fuzzy weights for solving hierarchical multi-criteria decision-making problems. A geometric mean-based equation and a logarithmic least square model are developed to elicit normalized acceptable triangular fuzzy multiplicative weights from an acceptable TFMPR. An equation and a linear program are established to transform triangular fuzzy multiplicative weights into triangular fuzzy additive weights. A geometric weighting-based equation and a pair of linear programs are devised to generate global priority fuzzy weights from local triangular fuzzy multiplicative weights. Three numerical examples, including a hierarchical multi-criteria decision-making problem, are used to illustrate how to apply the developed framework, and comparative studies are provided to show its availability and advantages.

1. Introduction

An analytic hierarchy process (AHP) is a commonly used multi-criteria decision making (MCDM) technique that has been diffusely applied to solving real-world problems [19,27,30]. Preferences provided by decision makers are expressed as ratios and organized as multiplicative comparison matrices (also called multiplicative preference relations (MPRs)) in the conventional AHP [34]. However, because many real decision problems involve uncertainty and complex information granularity [10,11,31,32], it is often an unrealistic scenario for a decision maker to elicit exact pairwise comparison ratios [27]. As such, different categories of preference relations have been proposed to characterize fuzzy pairwise comparisons, such as triangular fuzzy multiplicative preference relations (TFMPRs) [37], triangular fuzzy additive preference relations [38,44] and interval fuzzy additive preference relations [43,49]. In past decades, the triangular fuzzy AHP has been widely accepted and used in solving different MCDM problems [2–4,12,15,20–22,25,28,29,33,35,36,39,46,50].

* Corresponding author.
E-mail address: wangzj@xmu.edu.cn (Z.-J. Wang).

http://dx.doi.org/10.1016/j.ins.2017.03.028
0020-0255/

The consistency degree of a preference relation directly affects the rationality and reliability of the final decision result [17,24,34,42,45,47]. Saaty [34] presented a consistency index (CI) and a consistency ratio (CR) to measure the consistency degree of an MPR. Aguaron and Moreno-Jimenez [1] introduced another index called a geometric consistency index (GCI) and developed approximated GCI thresholds that match those of Saaty's CRs. Buckley [9] employed a triangular fuzzy approximated equation to introduce a consistency definition of TFMPRs. Dubois [17] showed the technical drawback of Buckley's consistency [9] and proved that this approximated transitivity equation holds if and only if a TFMPR is reduced to a consistent MPR. By using the consistencies of three constructed MPRs, Liu et al. [26] gave another consistency definition for TFMPRs. Wang [42] noted that Liu et al.'s consistency [26] has no invariance to permutations of the comparison object label names, and they proposed a new definition for consistent TFMPRs. The above existing research focuses on definitions and properties of consistent TFMPRs and tends to use Saaty's CR to measure the consistency degree of a TFMPR without examining its uncertainty level. However, a preference relation with high inconsistency and/or uncertainty indicates poor quality of decision input and could result in a misleading result [1,11,16,24]. To obtain a reliable decision result, it is necessary to consider both the inconsistency and the uncertainty degrees in the TFMPRs and to control these two degrees within their acceptable thresholds. This arrangement leads to our research on consistency and uncertainty measurements for TFMPRs

Priority weight elicitation is an important issue for decision making based on preference relations [8,19,27,48]. A host of priority methods have been devised to elicit triangular fuzzy weights from TFMPRs. For example, Van Laarhoven and Pedrycz [37] developed a logarithmic least square model (LLSM) to derive triangular fuzzy priorities of a TFMPR. Buckley [9] presented a triangular fuzzy geometric mean method to derive triangular fuzzy additive weights (TFAWs) of a TFMPR. Boender et al. [6] and Wang et al. [41] noted drawbacks of additive normalization methods in [7,13,14,18]. A modified LLSM was developed by Wang et al. [40] to directly derive a normalized TFAW vector from a TFMPR. Another common constraint used in normalizing crisp weights is multiplicative, i.e., the product of the normalized weights is equal to one. Crawford and Williams [16] proposed the row geometric mean method to obtain the normalized multiplicative weights of an MPR. Wang [42] introduced normalized constraints of triangular fuzzy multiplicative weights (TFMWs) and developed a geometric mean-based equation and an LLSM with an auxiliary constraint to generate a normalized TFMW vector from a TFMPR. It is observed from the aforementioned priority methods that the judgments in a comparison matrix are approximated as much as possible by the ratios of the generated priority weights from the interior/exterior direction or both interior and exterior directions. Because the support values of fuzzy judgments in a TFMPR are usually intervals, the uncertainty degrees of the triangular fuzzy weights generated by the existing fuzzy priority methods could be beyond an acceptable threshold even if the TFMPR has acceptability and the generated TFMWs or TFAWs are normalized. In addition, it is better to convert a TFMW vector into an equivalent TFAW vector if these TFMWs are regarded as the associated weights of a geometric weighting-based aggregation method. These results lead to our research on the elicitation of normalized acceptable TFMWs and the transformation between TFMWs and TFAWs.

This paper focuses on acceptability and priority elicitation for TFMPRs. We introduce geometric consistency and uncertainty indices to, respectively, measure consistency and uncertainty degrees in TFMPRs, and we propose the notion of acceptable TFMPRs. A key novelty of this acceptability notion is to account for both the inconsistency and uncertainty degrees in the TFMPRs. A highly uncertain TFMPR is determined to be unacceptable even if its inconsistency degree is low or it is consistent (see Example 1). We discuss the thresholds of acceptable consistency and develop some of the properties of acceptable TFMPRs. This paper proposes the notion of normalized acceptable TFMWs. A geometric mean-based equation and an LLSM are established to generate a normalized acceptable TFMW vector from an acceptable TFMPR. For solving hierarchical MCDM problems with TFMPRs, we develop an equation and a linear program to convert TFMWs into normalized TFAWs. A geometric weighting-based equation is devised to derive the modal value of a global TFMW, and a pair of linear programs is proposed to generate the support interval of a global TFMW from local TFMWs.

The remainder of this article is organized as follows. The next section furnishes the preliminaries on Saaty's CI and CR, GCIs of MPRs, and consistent TFMPRs. Section 3 proposes consistency and uncertainty measurements as well as acceptability checking for TFMPRs. A model is developed to elicit the normalized TFMWs of a TFMPR in Section 4 together with a transformation approach between TFMWs and TFAWs. Section 5 establishes an equation and a pair of linear programs to obtain global fuzzy weights from local TFMWs. Three illustrative numerical examples, including a hierarchical MCDM problem, are provided to demonstrate the developed framework in Section 6. Finally, Section 7 gives the main concluding remarks.

2. Preliminaries

Consider a decision-making problem with n objects that can be either criteria or alternatives denoted by the set $X = \{x_1, x_2, \ldots, x_n\}$. Using the pairwise comparison method, a decision maker elicits his/her preferences over the objects in X. In the conventional AHP, the preference values are provided by the decision maker under a bipolar ratio scale [1/S, S] with the neutral element of one and are organized by an MPR $A = (a_{ij})_{n\times n}$, where a_{ij} indicates a preference ratio of object x_i to x_j such that

$$1/S \le a_{ij} \le S, a_{ii} = 1, a_{ij}a_{ji} = 1, \quad i, j = 1, 2, \ldots, n. \tag{2.1}$$

Definition 2.1. [34] An MPR $A = (a_{ij})_{n\times n}$ is consistent if it satisfies multiplicative transitivity:

$$a_{ij} = a_{ik}a_{kj}, \quad i, j, k = 1, 2, \ldots, n. \tag{2.2}$$