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An algorithm for solving two-sided interval system of max-plus linear equations

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ABSTRACT

We study six types of solutions (weak, strong, tolerance, control, Left-localized, and Right-localized solutions) to a two-sided interval system of max-plus linear equations with the same vector of variables on both sides of the equations and obtain their corresponding solvability conditions. These conditions are in the form of two-sided systems of max-plus linear inequalities which can be derived as a union of a number of interval inclusion linear systems. Therefore, we could work on the interval inclusion linear systems, instead of directly solving these six types of solutions themselves. An optimization problem with the two-sided interval system of max-plus linear constraints may be solved using the convexity of each solution set of the interval inclusion linear systems.

1. Introduction

We provide a brief review of the max-plus linear systems in this section, in order to develop further a two-sided interval system of max-plus linear equations.

The main operations in any max-plus linear system are $\oplus$ and $\otimes$, where

$$u \oplus v = \max\{u, v\} \quad \text{and} \quad u \otimes v = u + v,$$

for any $u, v \in \mathbb{R}_{\max} = \mathbb{R} \cup \{-\infty\}$. These binary operations together with $\mathbb{R}_{\max}$, $(\mathbb{R}_{\max}, \oplus, \otimes)$, are called a ***max-plus algebra***. For numerous years, max-plus algebra has appeared under the name ***extremal algebra*** [8,38].

Cuninghame–Green [8] and Vorobjov [38] investigated the systems of max-plus linear equations when a vector of variables is on only one side of the equations. The system of inequalities in max-plus algebra were solved in [40]. References [2,8,14,16,33,42] used some key properties of max-plus algebra to analyze characteristics of max-plus linear discrete event systems including: production systems, queuing systems and array processors. In addition, Oldser [30] applied max-plus algebra to a closed network of railway connections in order to design a time schedule of trains running within the network. Recently, Seleim et al. [34] developed a manufacturing system modeling method using max-plus algebra for complex flow lines containing finite buffers and parallel identical stations.

A max-plus linear system is called a ***two-sided system of max-plus linear equations*** when it contains a vector of variables on both sides of equations. This system had been studied by many authors [3,4,9,39]. A general method for finding a solution can be found in [39]. Cuninghame–Green and Butkovič proposed a method in [9] that converges to a solution by

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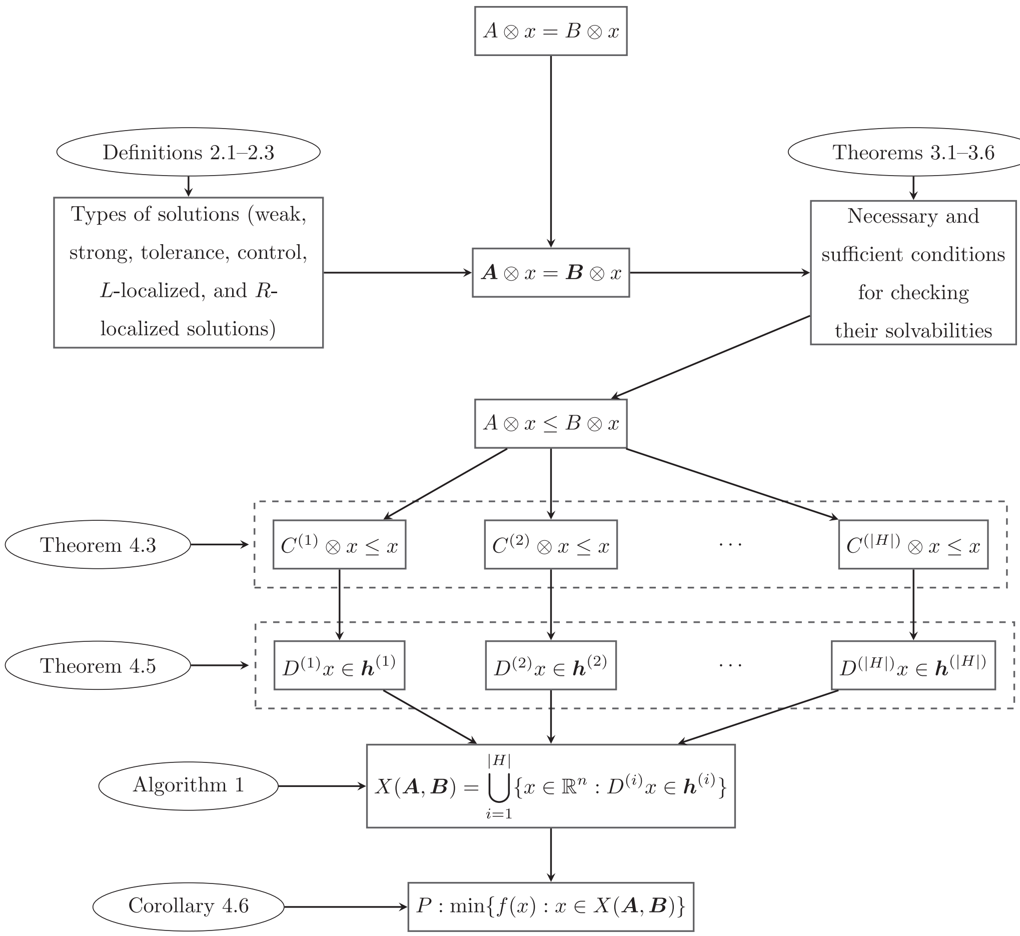

Fig. 1. Paper contributions at a glance.

generating pair-sequence of points from any finite starting point whenever a finite solution exists. Moreover, a method for finding all solutions of the two-sided systems was developed by Butkovič and Hegedüs in [3] which is analogous to that of solving systems of linear inequalities in the classical algebra. However, if a finite upper bound for all variables exists, Butkovič and Zimmermann [4] constructed an iterative approach to find the maximum vector solution smaller or equal to the finite upper bound vector. Butkovič and Zimmermann [4], under the finite upper bound assumption, were able to solve the problem by using homogeneity of max-plus linear functions.

Elements of the coefficient matrix and/or the right hand side vector of a linear system that are in interval form result in an interval system of linear equations. The theories of interval systems in the classical algebra can be found in [1,11,20,21,27]. In max-plus algebra, there are several types of solvability that have been studied based on the ideas of general algebra, that is, weak and strong solvabilities in [11,43], tolerance solvability in [12,22,24] and control solvability in [23]. A detailed discussion of interval solutions can be found in Chapter 6 of [11]. Recently, Left-localized and Right-localized solutions were investigated in [18,19]. In addition, Gavalec and Zimmermann [13] extended the basic concepts of one-sided systems of max-plus linear equations with interval coefficients from Cechlàrovà[5] and [6] to a general class of two-sided systems. All of the above articles also proposed necessary and sufficient conditions for checking their solvabilities.

This paper studies two-sided interval system of max-plus linear equations with the same vector of variables on both sides of equations. In Section 2, the concepts of weak, strong, tolerance, and control solutions of the two-sided interval system are defined in a similar fashion as in [13] together with two additional concepts of Left-localized and Right-localized solutions motivated by Leela-apiradee and Thipwiwatpotjana [18]. All solution types are illustrated in Example 2.4. In Section 3, necessary and sufficient conditions for checking the various solvabilities are presented. Theorems are stated in Section 4 to simplify the solution set of the two-sided interval system in the form of a union of sets containing solutions of interval inclusion linear systems. The conclusion of this article is provided in the last section. Before beginning the main ideas of this article, we provide a diagram showing the development of this paper in Fig. 1 and Table 1. The mathematical definition of each notation in the table will be stated before it is used.