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# State estimators for systems with random parameter matrices, stochastic nonlinearities, fading measurements and correlated noises

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ABSTRACT

Using the innovation analysis approach, the optimal linear state estimators, including the filter, predictor and smoother, in the linear minimum variance (LMV) sense are presented for a class of nonlinear discrete-time stochastic uncertain systems with fading measurements and correlated noises. Stochastic uncertainties of parameter matrices are depicted by correlated multiplicative noises. Stochastic nonlinearities are characterized by a known conditional mean and covariance. Different sensor channels have different fading measurement rates. The process and measurement noises are finite-step auto- and/or cross-correlated with each other. Two simulation examples verify the effectiveness of the proposed algorithms.



## 1. Introduction

In the last decade, research on random parameterized systems has attracted considerable attention due to their wide application to target tracking, industrial monitoring, communications, networks, and other areas [7]. In system modeling, stochastic uncertainties can be depicted by multiplicative noises in terms of random parameters [11]. In networked control systems (NCSs), stochastic phenomena such as transmission time delays and packet dropouts are almost unavoidable due to limited communication bandwidth and unreliable channels. These stochastic uncertainties induced by networks can be described by stochastic variables, which can be transformed into the system equations with random parameter matrices. Many estimation algorithms for such systems have been reported in the recent years; see e.g., [12,16,18,20,22,24,25] and references therein. Fading measurements are also frequently occurring phenomena in NCSs. They can reflect the degradation of communication channels or aging sensors. Missing measurements are one special case of fading measurements. Such systems have undergone considerable research; see e.g., [6,13,17,28]. Additionally, nonlinearities exist in almost all engineering systems [10,14,15,27]. To simplify modeling or processing, some minor nonlinear terms which have only a slight influence on the performance of systems are ignored. However, nonlinearities can severely degrade the performance of systems if they cannot be handled properly. Recently, stochastic nonlinearities have also gained much attention. These include state-dependent multiplicative noise as the special case; see e.g., [6,29].

In most estimation algorithms of stochastic systems, a general assumption is that process and measurement noises are uncorrelated. However, this is not always true in many practical applications such as a discretized continuous-time system

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[19], a normal system from the transformation of a singular system [21], a system measured in a common noisy environment, etc. The process and measurement noises can be finite-step auto- and/or cross-correlated in time. Recently, such systems have attracted much attention and many estimation algorithms have been presented [2–6,8,9]. The suboptimal Kalman-type recursive filter was designed for systems with correlated noises, random parameter matrices, multiple fading measurements and stochastic nonlinearities in [6]. The optimal linear estimators have been presented for systems with one-step correlated noises, stochastic nonlinearities, two-step transmission delays and packet dropouts in [29] and with finite-step correlated noises and packet dropout compensations in [23]. Similar solutions are also investigated for descriptor systems in [4]. However, one-step auto-correlated and/or two-step cross-correlated noises are only involved in [6,29,4], and random parameter matrices, stochastic nonlinearities and fading measurements are not considered in [23]. For systems with finite-step correlated noises, recursive Kalman-type filters were designed in [5,19]. However, they are suboptimal because their structures are fixed as with the Kalman recursive forms. Recently, an optimal filter in the minimum mean square error sense is developed in [8]. However, the fading measurements and stochastic nonlinearities are not considered and the multi-step predictors and smoothers are not designed. For multi-sensor systems with random parameter uncertainties and/or some random phenomena induced by networks, the centralized and distributed fusion state estimation algorithms in the least mean square sense are developed in [2,3,9,26]. However, only one-step auto-correlated and/or two-step cross-correlated noises are involved but stochastic nonlinearities are not taken into account.

Motivated by the above discussion, to the best of the authors' knowledge, the optimal linear estimation problem has not been fully resolved for systems with finite-step auto- and cross-correlated process and measurement noises, random parameter matrices, stochastic nonlinearities and fading measurements. Published results are either suboptimal or do not comprehensively consider the cases mentioned above. Moreover, most of them focus on the design of filters. In this paper, the aforementioned problems are considered fully. For example, random parameter matrices of the systems considered are correlated with each other at the same moment. The stochastic nonlinearities are characterized by a known conditional mean and covariance. Different sensor channels have different fading measurement rates. And process and measurement noises are finite-step correlated. For such a complex system, we present the optimal linear state estimation algorithms in the linear minimum variance sense, including filtering, prediction and smoothing, via an innovation analysis approach.

Notation: Standard notations are used throughout the paper. Superscript $\mathrm{T}$ denotes the transpose. $\mathrm{E}$ denotes the mathematical expectation. $\mathrm{tr}(\circ)$ denotes the trace of a matrix $\circ$. $\delta_{tk}$ is the Kronecker delta function. $I_n$ is an $n$ by $n$ identity matrix. $\perp$ denotes orthogonality. $\odot$ is the Hadamard product. $\mathrm{diag}(\circ)$ stands for a diagonal matrix whose diagonal elements consist of $\circ$. Prob[$\bullet$] represents the probability of the occurrence of the event $\circ$. $\hat{x}(\circ|\bullet)$ denotes the estimate of the stochastic variable $x(\circ)$ based on measurements taken before time $\bullet$, i.e., the projection of $x(\circ)$ on the linear space generated by the measurements taken before time $\bullet$. $\tilde{x}(\circ|\bullet) = x(\circ) - \hat{x}(\circ|\bullet)$ denotes the estimation error. $P_{xy}(\circ, *|\bullet)$ is the covariance matrix between estimation errors $\tilde{x}(\circ|\bullet)$ and $\tilde{y}(*|\bullet)$, with $P_{xx}(\circ, *|\bullet) = P_x(\circ, *|\bullet)$ and $P_{xy}(\circ, \circ|\bullet) = P_{xy}(\circ|\bullet)$. In an equation, the term $\{*\}$ represents the same as a list comprised of the front neighboring term.

The rest of this paper is organized as follows: In Section 2, the problem is formulated. In Sections 3–5, the optimal linear estimators including the filter, predictor and smoother are designed. In Section 6, two simulation examples are discussed. The study's conclusions are presented in Section 7.

## 2. Problem formulation

Consider a multi-channel discrete-time stochastic system with random parameter matrices, stochastic nonlinearities, fading measurements and correlated noises:

$$x(t+1) = A(t)x(t) + f(x(t), \xi(t)) + B(t)w(t) \tag{1}$$

$$y(t) = \gamma(t)C(t)x(t) + D(t)v(t) \tag{2}$$

where $x(t) \in R^n$ is the state vector to be estimated; $y(t) \in R^m$ is the measurement vector; $w(t) \in R^r$ is the process noise; $v(t) \in R^p$ is the measurement noise; $A(t)$, $B(t)$, $C(t)$ and $D(t)$ are random parameter matrices with suitable dimensions; $\gamma(t) = \mathrm{diag}(\gamma_1(t), \cdots, \gamma_m(t))$ describes the phenomena of fading measurements of different channels, where the scalar stochastic variable $\gamma_i(t)$ reflects the fading case of the $i$th measurement channel with a probability density function $p_i(t)$ over the interval $[0, 1]$ and known statistical properties $\mathrm{E}\{\gamma_i(t)\} = \bar{\gamma}_i(t)$ and $Cov\{\gamma_i(t)\} = \sigma_i^2(t)$, $i = 1, 2, \ldots, m$; $\gamma_i(t)$, $i = 1, 2, \ldots, m$ are independent of each other and of other random variables; the function $f(x(t), \xi(t))$ represents the stochastic nonlinearities of the state, where $\{\xi(t), t \geq 0\}$ is a zero-mean white noise sequence independent of $\{A(t), t \geq 0\}$, $\{B(t), t \geq 0\}$, $\{C(t), t \geq 0\}$, $\{D(t), t \geq 0\}$, $\{\gamma(t), t \geq 0\}$, $\{w(t), t \geq 0\}$ and $\{v(t), t \geq 0\}$.

The following assumptions will be used throughout the paper.

**Assumption 1.** The random parameter matrices $\{A(t), t \geq 0\}$, $\{B(t), t \geq 0\}$, $\{C(t), t \geq 0\}$ and $\{D(t), t \geq 0\}$ are correlated with each other at the same moment in time and are independent of $\{w(t), t \geq 0\}$ and $\{v(t), t \geq 0\}$. They have the following statistical properties:

$$\begin{aligned} \mathrm{E}\{M(t)\} &= \bar{M}(t), M = A, B, C, D; \\ Cov\Big\{M_{ij}(t), G_{su}(k)\Big\} &= T_{M_{ij}(t), G_{su}(t)}\delta_{tk}, M, G = A, B, C, D \end{aligned} \tag{3}$$

where $M_{ij}(t)$ and $G_{su}(k)$ are the $(i, j)$th and $(s, u)$th entries of matrices $M(t)$ and $G(k)$, respectively.