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Enhanced discrete-time Zhang neural network for time-variant matrix inversion in the presence of bias noises☆

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ABSTRACT

Inevitable noises and limited computational time are major issues for time-variant matrix inversion in practice. When designing a time-variant matrix inversion algorithm, it is highly demanded to suppress noises without violating the performance of real-time computation. However, most existing algorithms only consider a nominal system in the absence of noises, and may suffer from a great computational error when noises are taken into account. Some other algorithms assume that denoising has been conducted before computation, which may consume extra time and may not be suitable in practice. By considering the above situation, in this paper, an enhanced discrete-time Zhang neural network (EDTZNN) model is proposed, analyzed and investigated for time-variant matrix inversion. For comparison, an original discrete-time Zhang neural network (ODTZNN) model is presented. Note that the EDTZNN model is superior to ODTZNN model in suppressing various kinds of bias noises. Moreover, theoretical analyses show the convergence of the proposed EDTZNN model in the presence of various kinds of bias noises. In addition, numerical experiments including an application to robot motion planning are provided to substantiate the efficacy and superiority of the proposed EDTZNN model for time-variant matrix inversion.

1. Introduction

Matrix inversion is considered to be one fundamental problem widely encountered in science and engineering fields [1–5], such as optimization [1], robot control [2] and image processing [3]. In practice, the ability to invert matrix quickly and accurately determines the effectiveness of a computational tool [6]. Therefore, a lot of research efforts have been devoted to such a problem solving [7,8]. Matrix inversion, especially for large scale cases, has been significantly advanced and its time complexity has been much reduced in past years [6–9].

As a variant of constant matrix inversion, time-variant matrix inversion is becoming increasingly popular in recent years [10–13]. Generally speaking, time-variant matrix inversion is more complicated than constant matrix inversion, and the models for time-variant matrix inversion must satisfy the urgent requirement of real-time computation. Note that traditional methods for constant matrix inversion may not satisfy the real-time computational requirement of time-variant matrix inversion [14,15]. Specifically, the object matrix is varying with time, while time is inevitably consumed by each computational method. After the inverse of a time-variant matrix at a time instant is obtained, the matrix is not the original one.

With the characteristics of high-speed parallel processing and superiority in large scale online processing, neural networks have been widely employed in scientific computation and optimization [16–18]. Especially, recurrent neural networks (RNNs) have been presented and investigated as powerful alternatives to online scientific problems solving [19–21]. For solving online time-variant problems, including time-variant matrix inversion, Zhang neural network (ZNN), a special class of RNNs, was proposed [22]. Original Zhang neural network (OZNN) model is able to perfectly track time-variant solution for time-variant matrix inversion with

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the condition that the solving process is free of noises [10]. However, by considering that noises always exist and denoising may consume extra time, the pre-denoising method may not adapt well for online time-variant matrix inversion due to the urgent requirement of real-time computation in practice.

By considering the above situation, an enhanced Zhang neural network (EZNN) model is presented for time-variant matrix inversion [23]. Note that the EZNN model can perfectly track time-variant solution, and can suppress various kinds of bias noises, such as constant bias noise, bounded random bias noise and even (unbounded) linear-increasing bias noise, in one unified framework. Besides, as discrete-time models are convenient for numerical implementation on digital computers, the EZNN model and the OZNN model, which are continuous-time ZNN models, need to be discretized for practical realization [24–26]. Different from the OZNN model, the EZNN model contains an integral term. Thus, we may not obtain a convergent model by discretizing the integral term directly. To solve this problem, in this paper, new mathematical manipulations are employed to avoid discretizing the integral term directly. Then an enhanced discrete-time ZNN (EDTZNN) model is obtained for time-variant matrix inversion. Note that the EDTZNN model not only inherits the superiority of original discrete-time ZNN (ODTZNN) model [26] (i.e., the ODTZNN model can predict the inverse of time-variant matrix with high accuracy, or say, the inverse of time-variant matrix at a time instant can be obtained before/at that time instant), but also has the ability of suppressing various kinds of bias noises.

The remainder of this paper is organized into five sections. Section 2 presents the continuous-time ZNN models for time-variant matrix inversion, including the EZNN model and the OZNN model. In Section 3, by employing a new finite difference formula to discretize the EZNN model, the EDTZNN model is proposed. Moreover, the ODTZNN model is presented for comparison and the stability and convergence of EDTZNN model are analyzed. In Section 4, we analyze and investigate the EDTZNN model for time-variant matrix inversion in the presence of constant bias, bounded random bias noise and even linear-increasing bias noise. In Section 5, an application to robot motion planning is presented for further substantiating the efficacy of the EDTZNN model. Section 6 concludes the paper with final remarks. Before ending this section, it is worth pointing out here that the main contributions of this paper lie in the following facts.

- The EDTZNN model that is able to handle various kinds of bias noises is firstly proposed for time-variant matrix inversion.
- Theoretical analyses and results are presented to show the convergence of EDTZNN model.
- Numerical experiments including an application to robot motion planning are presented to substantiate the efficacy and superiority of the proposed model.

2. Continuous-time ZNN models

To solve the problem of time-variant matrix inversion, the following definition equation is considered:

$$A(t)X(t)=I, \quad \text{with } t\geq 0, \tag{1}$$

where $A(t)\in\mathbb{R}^{n\times n}$ is a nonsingular smoothly time-variant matrix, $X(t)\in\mathbb{R}^{n\times n}$ is the unknown matrix to be obtained, and $I\in\mathbb{R}^{n\times n}$ is the identity matrix. We assume that $A(t)$ and its time derivative are uniformly bounded.

To monitor and control the solving process of (1), we define the following matrix-valued indefinite error function:

$$E(t)=A(t)X(t)-I\in\mathbb{R}^{n\times n}. \tag{2}$$

Each element $e(t)$ of $E(t)$ should be convergent to zero. Based on error function (2) and original design formula $\dot{E}(t)=-\lambda E(t)$, where $\dot{E}(t)$ denotes the first order time-derivative of $E(t)$, we obtain the implicit OZNN model in the presence of bias noises as below [10]:

$$A(t)\dot{X}(t)=-\dot{A}(t)X(t)-\lambda(A(t)X(t)-I)+B(t), \tag{3}$$

where design parameter $\lambda>0$ and $B(t)\in\mathbb{R}^{n\times n}$ denotes matrix-valued noises, such as constant bias noise, bounded random bias noise and linear-increasing bias noise. If $B(t)=0$ (i.e., with no bias noise), implicit OZNN model (3) is evidently convergent [24]. Thus, we approximately have $X(t)=A^{-1}(t)$. Then, implicit OZNN model (3) can be rewritten as explicit form as below:

$$\dot{X}(t)=-X(t)\dot{A}(t)X(t)-\lambda X(t)(A(t)X(t)-I)+X(t)B(t). \tag{4}$$

Note that explicit OZNN model (4) is also Getz–Marsden (G–M) dynamic system (but with different origins) [10,27,28]. If $B(t)=0$, we obtain the OZNN model with no bias noise. Comparing explicit (4) with implicit (3), we have that implicit (3) is convergent globally, and that explicit (4) is convergent locally due to the approximation $X(t)=A^{-1}(t)$ [27]. Besides, model (4), or say, G–M dynamic system, cannot converge to the theoretical solution of problem (1) in the presence of bias noises.

To suppress bias noises, the new design formula

$$\dot{E}(t)=-\gamma_1E(t)-\gamma_2\int_0^t E(\tau)\mathrm{d}\tau \tag{5}$$

is employed [23], where design parameters $\gamma_1>0$ and $\gamma_2>0$. Based on design formula (5) and error function (2), the following implicit EZNN model in the presence of bias noises is obtained:

$$A(t)\dot{X}(t)=-\dot{A}(t)X(t)-\gamma_1(A(t)X(t)-I)-\gamma_2\int_0^t(A(\tau)X(\tau)-I)\mathrm{d}\tau+B(t). \tag{6}$$

Note that implicit model (6) can suppress various kinds of noises and compute the time-variant matrix inverse simultaneously. Besides, the following lemmas guarantee the convergence of implicit model (6).

Lemma 1. *With no bias noise, implicit EZNN model (6) converges to the theoretical solution of time-variant matrix inversion (1) with the continuous-time steady-state residual error (CTSSRE)* $\lim_{t\to\infty}\|A(t)X(t)-I\|_\mathrm{F}=0$*, where* $\|\cdot\|_\mathrm{F}$ *denotes the Frobenius norm of a matrix.*

Proof. See Appendix A for details.

Lemma 2. *In view of the continuous-time constant bias noise, implicit EZNN model (6) converges to the theoretical solution of (1) with the CTSSRE being 0.*

Proof. See Appendix B for details.

Lemma 3. *In view of the continuous-time bounded random bias noise, implicit EZNN model (6) converges to the theoretical solution of (1) with the CTSSRE being bounded and approximately in inverse proportion to* γ_1*.*

Proof. See Appendix C for details.

Lemma 4. *In view of the continuous-time (unbounded) linear-increasing bias noise, implicit EZNN model (6) converges to the theoretical solution of (1) with the CTSSRE being bounded and in inverse proportion to* γ_2*.*

Proof. See Appendix D for details.

Based on the above lemmas, we know that implicit model (6) is convergent, even in the presence of various kinds of bias noises. Thus, we approximately have $X(t)=A^{-1}(t)$. Then, implicit model