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Leader–follower consensus for multi-agent systems with three-layer network framework and dynamic interaction jointly connected topology

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ABSTRACT

This paper addresses the leader–follower consensus for linear and nonlinear multi-agent systems with three-layer network framework and dynamic interaction jointly connected topology. By introducing a novel concept of mirroring nodes for different roles of evolution agents, we remove the general assumptions that the topology among followers is specific and continuously contains a spanning tree, and propose a simple criterion to determine if a network is of three-layer framework. This criterion divides the agents of systems into leader-layer, middle-layer and follower-layer nodes according to their functions in cooperative behaviors. Moreover, by combining the state information of agents in different layers, two classes of linear and nonlinear leader–follower consensus protocols are designed with bounded consensus speed. In the sense of Lyapunov stability, it is proved that the leader–follower consensus for the closed-loop linear and nonlinear multi-agent systems can be achieved by employing novel error forms that decouple the leader–follower consensus errors of the three-layer network framework. Two simulation examples are presented to verify the proposed approach and demonstrate its effectiveness.

1. Introduction

Recent years witness a growing concern of the hot topic of the cooperative control of multi-agent systems due to its wide applications in sensor network, formation control, distributed control of multiple vehicles, complex networks, and so on [1–5]. For multi-agent systems, consensus is one of the most important and fundamental problems. Large numbers of significant results, including the single-integrator multi-agent systems, double-integrator multi-agent systems, and high-order-integrator multi-agent systems, were investigated in the past decades [6–8]. Additionally, factors such as disturbance and time-delay in networks were also studied [9–11]. However, some factors are not yet well addressed in the existing works and that limits the applications of multi-agent systems. For example, an important issue in consensus problem is to develop a unified framework, and based on this framework to develop the corresponding distributed controllers for linear and nonlinear multi-agent systems under dynamics interaction jointly connected topology, such that the state consensus of the whole group can be achieved based on the limited local information.

Today, many results of consensus behaviors have been established [6,8,12–16]. According to the numbers of leaders, consensus problem can be roughly divided into leaderless consensus (without a leader), leader–follower consensus (with one leader), and containment control (with multiple leaders) problems. In practice, there usually exists a real leader in real-world multi-agent systems to execute and regulate the actual operation instruction. Thus, it is significant to design the corresponding protocol for leader–follower consensus of multi-agent systems, in which topology analysis and protocol design are two critical tasks because they determine how the local behavior of each agent affects the group and the stability of multi-agent systems.

This paper addresses the leader–follower consensus from two aspects: network topology and the roles of agents. Generally speaking, leader–follower consensus problems were preliminary addressed in a static interaction topology [15,17,18]. In recent

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years, some works on dynamics interaction topology, including switching topologies, jointly connected topology, and intermittent communication topology, were reported [7,8,15,19]. In dynamics interaction topology, the control objectives were achieved by dynamics feedback controllers rather than static ones. Recently, Qin et al. addressed consensus with the underlying network and dynamics interaction topology [20], in which the topology switches arbitrarily among the possible weakly connected directed graphs with a certain restrictive assumption. Moreover, the consensus protocols in these aforementioned works were designed by combining different forms of agent's information and using appropriate stability analysis approaches [6,21,22]. Motivated by jointly connected topology, this paper studies the case of dynamic interaction jointly connected topology that is constructed by a network dynamical combination of leader agent and follower agents. Compared with the existing works, our topology condition is more practical and easier to be satisfied in dynamic networks of real systems. Moreover, no reported works are available on leader–follower consensus behaviors from the perspective of agent's roles in the network dynamics and the relationships among the agents. Note that different roles of agents will result in different allocations of resources.

According to the utilization of information in the protocol design and the consensus achievement process, as well as the two types of roles in the systems evolution, the follower agents can be divided into "communicate with each other" and "communicate with leader agent". To address the dynamics of different roles, we introduce a concept of mirroring node and propose a unified framework based on a three-layer network structure to analyze the leader–follower consensus behaviors. In the proposed framework, a mirroring node represents one role of an agent in evolution systems, which implies that the leader–follower consensus for all agents can be achieved if the consensus for each role is reached. For instance, if a follower agent plays two roles of "communicate with leader" and "communicate with other followers", then this follower agent will have two mirroring nodes in the three-layer network. This way, the effects of the different roles and the whole group dynamics on state consensus are decoupled. Specifically, the state consensus of the whole group can be regarded as the evolution of different roles of the agents. Following this idea, the proposed three-layer network framework is constructed by three types of nodes: (a) leader-layer nodes, which are the agents that evolve without being affected by any follower agents. The mirroring node of a leader-layer node is itself; (b) middle-layer nodes, which are the agents whose evolutions depend on the leader agent and some follower agents. Usually, these nodes have more than one mirroring nodes because they have different roles in the network and each role corresponds to a mirroring node; (c) follower-layer nodes, which are the agents whose evolutions are affected by all the follower agents. The mirroring node of a follower-layer nodes is itself because it has only one role.

The proposed three-layer network framework is able to integrate most of the existing network framework. For example, a node-node network framework [23] for the leader–follower consensus based on time-varying pinning links can be regarded as a special case of the three-layer network framework. Further, the proposed framework can be extended to neural network and multilayer predictive control such that decoupled modeling and control approaches can be developed. Additionally, dynamics interaction jointly connected topology in leader–follower consensus of multi-agent systems with dynamical framework has not been fully investigated, which motivates our current study.

Based on the above considerations, this paper addresses the leader–follower consensus from two aspects: (1) network topology is dynamics interaction jointly connected; and (2) the state consensus of the whole group is realized by different types of agents. Compared with the material [24], our main contributions are listed as follows: (1) propose a novel network framework constructed by three types of roles of the agents to analyze the evolution behaviors of systems; (2) design a comprehensive protocol to realize the leader–follower consensus for linear multi-agent systems; and (3) extend the results to the design of a nonlinear protocol for the nonlinear multi-agent systems. Our results show that leader–follower consensus for closed-loop linear and nonlinear multi-agent systems with three-layer network framework and dynamics interaction jointly connected topology can be achieved if control parameters are properly selected.

The outline of the paper is as follows: Section 2 introduces the preliminaries, problem formulation, lemmas and definitions. Section 3 develops the main results. Section 4 presents some simulation results, which are followed by concluding remarks in Section 5.

2. Problem statement

2.1. Graph theory

This paper considers the problem of leader–follower consensus with dynamic interaction jointly connected topology. Three classes directed graphs are introduced: (1) The topology of an N-agent system can be described by a directed graph $\mathcal{G}(t)=(\mathcal{V},\mathcal{E}(t),\mathcal{A}(t))$, where $\mathcal{V}=\{1,\dots,N\}$ is the set of vertices representing the set of the agents, $\mathcal{E}(t)\in\mathcal{V}\times\mathcal{V}$ is the set of edges, and $\mathcal{A}(t)=[a_{ij}(t)]_{N\times N}$ is the weighed adjacency matrix that has non-negative elements $a_{ij}(t)$. A directed path from node i to node j is a finite ordered sequence of edges, $(i,k_1),(k_1,k_2),\dots,(k_p,j)$ with distinct nodes $k_q, q=1,\dots,p$. Note that $a_{ij}>0$ if and only if $(i,j)\in\mathcal{E}(t)$ and $a_{ij}=0$ otherwise. Let $\mathcal{L}(t)$ be the Laplacian of the directed graph $\mathcal{G}(t)$ consisting of N agents. The Laplacian matrix of $\mathcal{G}(t)$, $\mathcal{L}(t)=[l_{ij}(t)]_{N\times N}$, satisfies $l_{ij}(t) = -a_{ij}(t)$ if $i\neq j$; otherwise $l_{ii}(t)=\sum_{k=1,k\neq i}^{N}a_{ik}(t)$. Furthermore, if there exists a node so that there is at least one directed path from this node to any other node, the graph is said to contain a directed spanning tree and this node is called the root. (2) The dynamic interaction topology consists of $(N+1)$ nodes and is denoted as a derivative directed graph $\overline{\mathcal{G}}(t)=(\overline{\mathcal{V}},\overline{\mathcal{E}}(t),\overline{\mathcal{A}}(t))$, which is generated by the directed graph $\mathcal{G}(t)$ of follower agents and the leader matrix $\mathcal{D}(t)=diag\{d_1(t),\dots,d_N(t)\}$ that represents the varying communication path between follower agents and leader agent. If follower agent i can obtain the information from the leader at time $t\geq 0$, then $d_i(t)=1$; otherwise, $d_i(t)=0$. (3) The dynamic interaction jointly connected topology consists of $(N+1)$ nodes and is denoted as a union of derivative directed graph $\overline{\mathcal{G}}_{un,i}(t)$, which is given as $\cup_{i=1}^{s}\overline{\mathcal{G}}_i=\{\overline{\mathcal{V}},\cup_{i=1}^{s}\overline{\mathcal{E}}_i(t),\cup_{i=1}^{s}\overline{\mathcal{A}}_i(t)\}$, $s\in\mathbb{N}$.

Specifically, based on the fact that agents play different roles in the evolution of the system, a set of mirroring nodes $\tilde{\mathcal{V}}=\{\tilde{\mathcal{V}}_1,\tilde{\mathcal{V}}_2,\dots\}$ is introduced to represent the roles of all agents. The mirroring networks of $\overline{\mathcal{G}}(t)$ are defined as $\tilde{\mathcal{G}}(t)=\{\overline{\mathcal{V}}\cup\tilde{\mathcal{V}},\overline{\mathcal{E}}(t)\cup\tilde{\mathcal{E}}(t),\overline{\mathcal{A}}(t)\cup\tilde{\mathcal{A}}(t)\}$, $s\in\mathbb{N}$, where $\tilde{\mathcal{V}}$ is a mirroring set of nodes determined by $\overline{\mathcal{V}}$, $\tilde{\mathcal{E}}(t)$ and $\tilde{\mathcal{A}}(t)$ are corresponding time-varying mirroring sets of edges and mirroring weighted adjacency matrices, respectively. Before moving on, the following lemmas are introduced.

Lemma 1 (Ren *[12]*). *Let $\overline{\mathcal{L}}(t)$ be a structure matrix of derivative directed graph $\overline{\mathcal{G}}(t)$. Then, $\overline{\mathcal{G}}(t)$ has a spanning tree if and only if $\overline{\mathcal{L}}(t)$ has a nonzero eigenvalue with multiplicity 1 and all the other nonzero eigenvalues are with positive real parts.*

Lemma 2 (Wang et al. *[8]*). *Suppose that a matrix $A=[a_{ij}]\in\mathbb{R}^{n\times n}$ satisfies $a_{ij}\leq 0$ for any $i\neq j$. Then, the following statements are equivalent:*

(1) *A is a nonsingular M-matrix.*