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Mittag–Leffler stability analysis on variable-time impulsive fractional-order neural networks

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Mittag–Leffler stability analysis of fractional-order neural networks with variable-time impulses is addressed in this paper. Several well-proposed conditions with theoretical demonstration ensuring that every solution of concerned models intersects each surface of the discontinuities exactly once are provided. Meanwhile, by applying B-equivalence method, the reduced fractional-order neural networks with fixed-time impulsive effects can be regarded as the comparison systems of the investigated original network models. Furthermore, a series of sufficient criteria illustrating the same stability properties between both variable-time impulsive fractional-order neural networks and the fixed-time alternative, and guaranteeing the stability of the considered models are presented. Finally, two simulation examples are presented to demonstrate the efficiency and feasibility of the achieved results.

1. Introduction

In recent several decades, owing to the extensive applications in image processing, pattern recognition, associative memory, classification of patterns, solving linear and nonlinear programming, fixed-point computation, and other areas, neural networks have received considerable attention, and various issues on its dynamical behaviors have been reported, see [1–6] and references therein.

As we all know, there are many real-world systems and natural processes display certain dynamic behaviors with both continuous and discrete characteristics [7]. For instance, in implementation of electronic networks, automatic control systems, telecommunication systems and artificial intelligence networks, the system states are very likely subject to instantaneous perturbations and experience abrupt changes at certain instants [8]. Apparently, neither pure continuous or pure discrete models could describe these familiar impulsive phenomena, thereupon, people introduced a sort of impulsive dynamical systems which are composed of three parts: continuous-time subsystem, discrete subsystem, and a switching rule which determines the impulse moments [9].

Generally speaking, the impulsive systems can be classified as fixed-time impulsive systems and variable-time impulsive systems based on different switching rules. During the last few decades, numerous monographs and articles focused on the stability analysis for neural networks with fixed-time impulses, see [10–19] and references therein. In the meantime, there exist some literatures paying attention to dynamics for variable-time impulsive neural networks. It should be noted that the impulse moments of the variable-time impulsive systems are not prescribed, and will not clear until one starts to look for a certain solution [20–23]. Mostly, as far as we know, the existing stability results on variable-time impulsive neural networks are proved by utilizing comparison system method, and the established comparison systems yet also be variable-time impulsive systems but with one dimension. Therefore, difficulties will still be encountered when analyzing the dynamics for such comparison systems.

Recently, in Akhmet's monograph [24], a powerful analytical tool of B-equivalence was developed, which has strong superiority for disposing systems with solutions that have discontinuities at variable moment of time, for it could reduce such variable-time impulsive systems to fixed-time impulsive systems. The reduced systems can be regarded as the comparison systems of the original ones. In [25,26], the authors borrowed the B-equivalence method and established some sufficient conditions of dynamic behaviors for variable-time impulsive neural networks.

It is recognized that fractional-order models have showed more merits than classical integer-order models in describing the hereditary and memory properties for various materials and processes. Fractional-order models have demonstrated a crucial role

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in modeling and analyzing phenomena in various fields of science and engineering such as viscoelastic systems, quantitative finance, diffusion waves, mechanics, electromagnetism, heat transfer, electrical circuits, system identification and propagation [27–29]. As a consequence, fractional calculus has been incorporated into the classical integer-order neural networks, and a considerable number of articles on dynamics of fractional-order neural networks have been published, for example, see [30–33] and references therein.

Nowadays, the impulsive fractional-order system has been a heated topic, and more and more impulsive dynamical results on fractional-order models have been reported. In [34], by using the comparison principle, coupled with the Lyapunov function method, the authors investigated stability analysis of the zero solution for a class of impulsive fractional functional differential systems. In [35], the authors established some sufficient criteria for the Mittag–Leffler stability and asymptotic stability of impulsive fractional-order systems based on Lyapunov direct method and comparison principles. In [36], by applying the fractional Lyapunov method and Mittag–Leffler functions, the authors presented several sufficient conditions of global Mittag–Leffler stability and the synchronization for fractional chaotic networks via non-impulsive linear controller. In [37], some sufficient conditions were derived to ensure the existence, uniqueness and the asymptotic stability of the fractional-order neural networks on Riemann–Liouville's sense. In [38], the authors discussed the feedback control systems of impulsive fractional differential equations on networks, and a systematic method for constructing a global Lyapunov function for such models was provided by combining some graph theory and the Lyapunov method. As far as we know, there exists no published result for impulsive fractional-order models on the basis of B-equivalence method.

Motivated by the aforementioned discussions, we consider the Mittag–Leffler stability analysis for variable-time impulsive neural networks by utilizing the B-equivalence method. In the first place, some assumptions ensuring each solution of the proposed models that intersects each surface of the discontinuity exactly once will be listed and the corresponding proofs will be given. In the next place, by constructing a B-map the original systems will reduce to impulsive fixed-time ones, furthermore, demonstration showing the same stability properties between the variable-time impulsive fractional-order neural networks and the corresponding models with fixed-time impulses will be provided. At last, several sufficient criteria guaranteeing the Mittag–Leffler stability for the addressed networks by means of the presented comparison systems will be established.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times n$ real matrices. I means the identity matrix with appropriate dimension. P^T stands for the transpose of matrix P. $\|x\|$ denotes the Euclidean norm of a real vector x. For a real matrix A, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ stands for the maximal and minimal eigenvalue of A, respectively. $\|A\| = \sqrt{\lambda_{max}(A^TA)}$ denotes the spectral norm of matrix A. $A>0$ (<0) means the symmetric matrix A is positive definite (or negative definite).

2. Model description and preliminaries

Let $\mathbb{R}_+=[0,+\infty)$, $\mathbb{Z}_+=\{1,2,3,\dots\}$, $G_i=(t_{i-1},t_i)\times\mathbb{R}^n, i=1,2,3\dots$, and $G=\bigcup_{i=1}^{\infty}G_i$. We further denote $\Gamma_i=\{(t,x(t))\in\mathbb{R}_+\times G : t=\theta_i+\tau_i(x(t)), t\in\mathbb{R}_+, x\in G, G\subset\mathbb{R}^n\}$ the ith surface of discontinuities, and the sequence $\theta=\{\theta_i\}_{i=1}^{\infty}$ satisfies $0=\theta_0<\theta_1<\theta_2<\dots<\theta_i<\theta_{i+1}<\dots$ with $\lim_{i\to+\infty}\theta_i=\infty$.

Consider the following fractional-order neural networks with variable-time impulses:

$$\begin{cases} D^{\alpha}_{t_0,t}x(t) = -Cx(t)+Af(x(t)), \\ \Delta x|_{t=\theta_i+\tau_i(x)} = J_i(x), \end{cases} \tag{1}$$

with the following fractional-order neural networks as its continuous subsystem:

$$D^{\alpha}_{t_0,t}x(t) = -Cx(t)+Af(x(t)), \tag{2}$$

and the state jumps as its discrete subsystem:

$$\Delta x|_{t=\theta_i+\tau_i(x)} = J_i(x), \tag{3}$$

where $D^{\alpha}_{t_0,t}$ denotes the operator of α-order $(0<\alpha<1)$ Caputo fractional derivative with t_0 and t, respectively, the lower limit and the upper limit. $x\in G\subset\mathbb{R}^n$ stands for the state vector of impulsive neural networks, $C=diag(c_1,c_2,\dots,c_n)>0$ the decay rate matrix, $A=(a_{ij})_{n\times n}\in\mathbb{R}^{n\times n}$ the connection weight matrix, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\dots,f_n(x_n(t)))^T$ the activation function vector with $f(0)=0$. $J_i(x)$ are continuous, and $J_i(0)=0$. $\Delta x|_{t=\xi_k}=x(\xi_k^+)-x(\xi_k)$ with $x(\xi_k^+)=\lim_{t\to\xi_k+0}x(t)$ stands for the state jump at moment ξ_k, satisfying $\xi_k=\theta_k+\tau_k(x(\xi_k))$. Without loss of generality, we assume that the solution $x(t)$ is left continuous at impulse point, that is, $x(\xi_k^-)=\lim_{t\to\xi_k-0}x(t)=x(\xi_k)$.

Remark 1. The existence and uniqueness of solutions for fractional differential equations with fixed-time impulses has been investigated by several authors, see Refs. [39,40]. On account of this, in this paper, we would discuss the stability of zero solution for systems (1).

In the following, we will recall some necessary definitions.

Definition 1 (*Podlubny [27]*). For any $t\ge t_0$, the Riemann–Liouville fractional integral of order α with the lower limit t_0 for a function f defined by

$$I^{\alpha}_{t_0,t}f(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-s)^{\alpha-1}f(s)ds, \quad n-1<\alpha<n, \tag{4}$$

where $I^{\alpha}_{t_0,t}$ denotes the fractional integral operator, and $\Gamma(\cdot)$ means the Gamma function operator.

Definition 2 (*Podlubny [27]*). For any $t\ge t_0$, the Caputo fractional derivative of order α with the lower limit t_0 for a function $f\in C^n[[t_0,t],\mathbb{R}^n]$ is defined as

$$D^{\alpha}_{t_0,t}f(t)=I^{n-\alpha}_{t_0,t}\left[\frac{d^n}{dt^n}f(t)\right]=\frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}(t-s)^{n-\alpha-1}f^{(n)}(s)ds, \quad n-1<\alpha<n. \tag{5}$$

Remark 2. When $0<\alpha<1$, $D^{\alpha}_{t_0,t}x(t)=\frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}(t-s)^{-\alpha}x'(s)ds$, and it holds that $I^{\alpha}_{t_0,t}D^{\alpha}_{t_0,t}x(t)=x(t)-x(t_0)$.

Definition 3 (*Podlubny [27]*). The one-parameter Mittag–Leffler function is defined as

$$E_{\alpha}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}, \quad \alpha>0,\ z\in\mathbb{C}. \tag{6}$$

The two-parameter Mittag–Leffler function also appears frequently and has the following form

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)}, \quad \alpha>0,\ \beta>0,\ z\in\mathbb{C}. \tag{7}$$

When $\beta=1$, one has $E_{\alpha}(z)=E_{\alpha,1}(z)$. Also, $E_{1,1}(z)=e^z$. Moreover, the Laplace transform of the two-parameter Mittag–Leffler function is

$$\mathcal{L}[t^{\beta-1}E_{\alpha,\beta}(-\lambda t^{\alpha})]=\frac{s^{\alpha-\beta}}{s^{\alpha}+\lambda}, \quad t\ge 0,\ \Re(s)>|\lambda|^{\frac{1}{\alpha}}, \tag{8}$$

where t and s are the variables in the time domain and Laplace