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Delay-partitioning approach design for stochastic stability analysis of uncertain neutral-type neural networks with Markovian jumping parameters☆

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ABSTRACT

This paper investigates the problem of stability analysis for uncertain neutral-type neural networks with Markovian jumping parameters and interval time-varying delays. By separating the delay interval into multiple subintervals, a Lyapunov–Krasovskii methodology is established, which contains triple and quadruple integrals. The time-varying delay is considered to locate into any subintervals, which is different from existing delay-partitioning methods. Based on the proposed delay-partitioning approach, a stability criterion is derived to reduce the conservatism. Numerical examples show the effectiveness of the proposed methods.

1. Introduction

In recent decades, delayed neural networks have obtained much more considerable attention because they often exist in a lot of areas, such as signal processing, model identification and optimization problem [1–5]. Since that time delays is frequently encountered, the problem of stability analysis of time delayed neural networks is an important point in the field of neural networks recently. Besides, many important results have been proposed in [6–11]. Furthermore, the neutral-type systems have been always investigated, because that the past state of the network affects on the current state. Due to the existence of parameter variations, modeling errors and process uncertainties, stability analysis of neutral uncertainties systems has gained a lot of attention [12–14].

In addition, Markovian jump systems can be described by a set of linear systems with the transitions during models determined by a Markovian chain in a finite mode set. This system has been applied in economic systems, modeling production systems and other practical systems. Markovian jump systems may encounter random abrupt variations in their structures when the time goes by. For this kind of systems, we refer readers to [15,16].

Delay-partitioning, which divides delay interval into subintervals, could obtain less conservative stability conditions [17]. After that the delay-partition idea was firstly proposed in [17], many researchers have focused on designing delay-partition technologies [18–23]. For example, the reference [23] considered a delay partitioning approach to delay-dependent stability analysis for neutral type neural networks. Moreover, some researchers in [20,21] have improved the idea to analyze the stability of time-varying delays.

This paper investigates the problem of stochastic stability analysis for neutral-type uncertain neural networks with Markovian jumping parameters and time-varying delays. A novel Lyapunov–Krasovskii function that involves triple and quadruple integrals terms is constructed to obtain less conservative stability conditions. The proposed delay-partitioning method divide the time-delay interval $[\tau_2^-, \tau_2^+]$ finely by introducing the time variable $\rho(t)$. According to $[(k-1)h_2, kh_2] = [(k-1)h_2, (k-1)h_2+\rho(t)]+[(k-1)h_2+\rho(t), kh_2]$, the time-delay $\tau_2(t)$ will determine on the certain subintervals any further. Moreover, the distributed delay is considered as time-varying delay $\int_{t-\tau_3(t)}^{t} f(x(s))ds$. The time variable $\rho_1(t)(\rho_1(t)=\frac{\tau_3(t)}{q})$ is introduced to cope with the delay-partitioning problem. A new integral inequality is applied in terms of reciprocally convex inequality, in order to ensure conservatism reduction. Less conservative stability criteria are

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evaluated by using linear matrix inequalities. Numerical examples are given to demonstrate the effectiveness of the proposed methods.

Notations: $\Re^n$ denotes n-dimensional Euclidean space and $\Re^{n\times n}$ is the set of all $n\times n$ real matrices. For symmetric matrices X, $X>0$ $(X\geq 0)$ represents that it is a real symmetric positive definite matrix (positive semi-definite). For symmetric matrices X and Y, $X>Y(X\geq Y)$ means that the matrix $X-Y$ is positive definite (nonnegative). A^T stands for the transpose of matrix A. $sym(A)$ denotes $A+A^T$. $*$ denotes the elements below the main diagonal of a symmetric block matrix. I means the identity matrix with appropriate dimensions. $O_{m\times n}$ represents zero matrix with $m\times n$ dimensions. $\|\cdot\|$ denotes the Euclidean norm of vector and its induced norm of matrix. (Ω, F, P) is a complete probability space with a filtration $\{F_t\}_{t\geq 0}$, in which Ω, F and P separately denote a sample space, the σ-algebra of sunset of sample space and the probability measure on F. $L^p_{F_0}([-\tau,0];R^n)$ is the family of all F_0-measurable $C([-\tau,0];R^n)$-valued-random variables $\xi=\xi(\theta):-\tau\leq\theta\leq 0$ such that $\sup_{-\tau\leq\theta\leq 0}E\{\|\xi(\theta)\|_2^2\}<\infty$, where $E\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. $col[x_1,x_2,\ldots,x_n]$ means $[x_1^T,x_2^T,\ldots,x_n^T]^T$.

2. Preliminaries

Let $\{r_t, t\geq 0\}$ be right-continuous Markov process on the probability space which takes values in the finite space $S=\{1,2,\ldots,N\}$ with generator $\pi=(\pi_{ij})(i,j\in S)$ also called jumping transfer matrix given by

$$P\{r_{t+\Delta}=j\,|\,r_t=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta) & \text{if } j\neq i,\\ 1+\pi_{ii}\Delta+o(\Delta) & \text{if } j=i,\end{cases} \tag{1}$$

where $\Delta>0$ and $\lim_{\Delta\to 0}\frac{o(\Delta)}{\Delta}=0$. $\pi_{ij}\geq 0$ is the transition rate from i to j if $i\neq j$ and $\pi_{ii}=-\sum_{j\neq i}\pi_{ij}$. Consider a class of neutral-type uncertain neural networks with Markovian jumping parameters and mixed delays

$$\dot y(t)=E(r_t,t)\dot y(t-\tau_1(t))+A(r_t,t)y(t)+B(r_t,t)g(y(t))+C(r_t,t)g(y(t-\tau_2(t)))+D(r_t,t)\int_{t-\tau_3(t)}^{t}g(y(s))ds+I, \tag{2}$$

where $y(t)=[y_1(t),\ldots,y_n(t)]^T\in R^n$ is the neuron state vector; $g(y(\cdot))=[g_1(y_1(\cdot)),\ldots,g_n(y_n(\cdot))]^T\in R^n$ is the neuron activation function vectors; $I=[I_1,\ldots,I_n]$; we shall use the following assumption.

Assumption 2.1. For the uncertain matrices of the system (2), $E(r_t,t), A(r_t,t), B(r_t,t), C(r_t,t), D(r_t,t)$ denote interconnection weight matrices and can be described by

$$\begin{aligned}&\begin{bmatrix}E(r_t,t) & A(r_t,t) & B(r_t,t) & C(r_t,t) & D(r_t,t)\end{bmatrix}\\&=\begin{bmatrix}E(r_t) & A(r_t) & B(r_t) & C(r_t) & D(r_t)\end{bmatrix}\\&\quad+Y(r_t)I(r_t,t)\begin{bmatrix}M_e(r_t) & M_a(r_t) & M_b(r_t) & M_c(r_t) & M_d(r_t)\end{bmatrix},\end{aligned} \tag{3}$$

where $A(r_t)=-diag(a_1(r_t),\ldots,a_n(r_t))<0$, $B(r_t)$, $C(r_t)$, $D(r_t)$, $E(r_t)$, $Y(r_t)$, $M_a(r_t)$ $M_b(r_t)$, $M_c(r_t)$, $M_d(r_t)$, $M_e(r_t)$ represent the constant matrices. In order to simplify the notation, let $r_t=i$. Then, one has $A(r_t)=A_i$, $B(r_t)=B_i$, and so on. $I(r_t,t)$ is an unknown time-varying matrix with Lebesgue measurable elements bounded by $I^T(r_t,t)I(r_t,t)\leq I$, which is the identity matrix of appropriate dimensions.

Assumption 2.2. The time delays $\tau_1(t)$, $\tau_2(t)$, $\tau_3(t)$ are continuously time-varying functions in (2) that satisfy

$$\begin{cases}0\leq\tau_1^-\leq\tau_1(t)\leq\tau_1^+, & \dot\tau_1(t)\leq\mu_1,\\ 0\leq\tau_2^-\leq\tau_2(t)\leq\tau_2^+, & \dot\tau_2(t)\leq\mu_2,\\ 0\leq\tau_3(t)\leq\tau_3^+.\end{cases} \tag{4}$$

For any integers $m\geq 1$, $l\geq 1$, $q\geq 1$, let $h_1=\frac{\tau_2^-}{m}$, $h_2=\frac{\tau_2^+-\tau_2^-}{l}$, $h_3=\frac{\tau_3^+}{q}$, $\rho_1(t)=\frac{\tau_3(t)}{q}$, $\rho(t)=\frac{\tau_2(t)-\tau_2^-}{l}$, then $[0,\tau_2^-]$ can be divided into m segments, $[\tau_2^-,\tau_2^+]$ can be divided into l segments, $[0,\tau_3(t)]$ can be divided into q segments. $[0,\tau_2^-]=\bigcup_{i=1}^m[(i-1)h_1, ih_1]$, $[\tau_2^-,\tau_2^+]=\bigcup_{i=1}^l[\tau_2^-+(i-1)h_2,\tau_2^-+ih_2]$, $[0,\tau_3^+]=\bigcup_{i=1}^q[(i-1)h_3, ih_3]$, $[0,\tau_3(t)]=\bigcup_{i=1}^q[(i-1)\rho_1(t), i\rho_1(t)]$. For each subinterval $[(k-1)h_2, kh_2], k=1,2,\ldots,l$, it easy to obtain $[(k-1)h_2,kh_2]=[(k-1)h_2,(k-1)h_2+\rho(t)]\cup[(k-1)h_2+\rho(t),kh_2]$. On the other hand, for any $t\geq 0$, there exists an integer $k\in\{1,2,\ldots,l\}$ such that $\tau_2(t)\in[(k-1)h_2,kh_2]$.

Assumption 2.3. Each activation function $f_i(\cdot)$ in the system (2) is continuous and bounded, which satisfies the following inequalities

$$\sigma_i^-\leq\frac{g_i(a)-g_i(b)}{a-b}\leq\sigma_i^+,\quad k=1,2,\ldots,n,\ \text{and } g_i(0)=0, \tag{5}$$

where $a, b\in\Re$, $a\neq b$, σ_i^-, σ_i^+ are known constants.

Remark 2.1. σ_i^-, σ_i^+ $(i=1,2,\ldots,n)$ are some constants, and they can be positive, negative, and zero in Assumption 2.3. Consequently, this type of activation function is clearly more general than both the usual sigmoid activation function and the piecewise linear function $g_i(u)=\frac{1}{2}(|u_{i+1}|-|u_i|)$, which is useful to get less conservative result.

Let the equilibrium point y^* in (2) be the origin, by setting $x(t)=y(t)-y^*$. The system (2) can be rewritten in the following form

$$\dot x(t)=E(r_t,t)\dot x(t-\tau_1(t))+A(r_t,t)x(t)+B(r_t,t)f(x(t))+C(r_t,t)f(x(t-\tau_2(t)))+D(r_t,t)\int_{t-\tau_3(t)}^{t}f(x(s))ds, \tag{6}$$

where $x(t)=[x_1(t),\ldots,x_n(t)]^T\in R^n$ is the neuron state vector, $f_i(x(\cdot))=g_i(x_i(\cdot)+y_i^*)-g_i(y_i^*)$ $i=1,2,\ldots,n$. From Assumption 2.3, the transformed neuron activation function satisfies

$$\sigma_i^-\leq\frac{f_i(a)-f_i(b)}{a-b}\leq\sigma_i^+,\quad k=1,2,\ldots,n,\ \text{and } f_i(0)=0. \tag{7}$$

The purpose of this paper is to obtain the stability theorem for the system (6). The following robustly stochastically stable definition is introduced.

Definition 2.1 (*[20]*). The trivial solution (equilibrium point) of the neutral-type neural networks with Markovian jumping parameters (2) is said to be robustly stochastically stable in the mean square, if $\lim_{t\to\infty}E\{\|x(t)\|^2\}=0$, for all admissible uncertainties satisfying (3).

Before deriving the main results, the following lemmas which will be used are given.

Lemma 2.1 (*[24]*). *For any constant matrices V, $W\in\Re^{n\times n}$ with $M>0$, scalars $b>a$, vector function $V:[a,b]\to\Re^m$ such that integrations in the following are well-defined, then*

$$(b-a)\int_a^b V^T(s)MV(s)ds\geq\left(\int_a^b V(s)ds\right)^T M\int_a^b V(s)ds, \tag{8}$$

$$\frac{\tau^2}{2}\int_{-\tau}^{0}\int_{t+\theta}^{t}W^T(s)MW(s)dsd\theta\geq\left(\int_{-\tau}^{0}\int_{t+\theta}^{t}W(s)dsd\theta\right)^T M\int_{-\tau}^{0}\int_{t+\theta}^{t}W(s)dsd\theta. \tag{9}$$

Lemma 2.2 (*[25]*). *For the given matrices $Q=Q^T$, D, E with appropriate dimensions, one can conclude that*

$$Q+DF(t)E+E^TF^T(t)D^T<0, \tag{10}$$