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Exponential stabilization of neural networks with time-varying delay by periodically intermittent control

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ABSTRACT

This paper investigates the exponential stabilization of neural networks with time-varying delay by periodically intermittent control. By employing the free-matrix-based integral inequality and using some new analysis techniques, some novel exponential stabilization criteria are derived based on the Lyapunov–Krasovskii (L–K) functional method. The obtained criteria are in terms of linear matrix inequalities without transcendental equation, instead of nonlinear matrix inequalities, which reduces the computational burden. Compared to existing results in corresponding literatures, our results have a wider range of applications, and overcome no feasible solution if the information on the sizes of delays is ignored for the design of the intermittent controller. A numerical simulation is provided to show the effectiveness and the benefits of the theoretical results.

1. Introduction

In past decades, neural networks have been extensively investigated and rapidly developed since they have wide applications in a variety of fields, such as secure communication, quantum devices, pattern classification, associative memory, etc. Due to the inherent characteristics of neural networks, they may have undesirable dynamical behaviors such as chaos, oscillators and instability. However, it is essential that the designed neural networks are stable in their applications. Thus, the stability analysis and control of neural networks have attracted significant attentions, see, e.g., [1–16] and references therein. Up until now, many control strategies have been put forward to realize stabilization and synchronization for the considered systems, including sampled-data control [17], L_2 control [18], H_∞ control [19,20], quantized control [21], adaptive control [22], impulsive control [23], intermittent control [24–27] and so on.

The type of control concerned in this paper is intermittent control, which was proposed in the seminal paper of Craik (see [28]) and has aroused a great deal of interest due to its broad potential applications [29–33]. Compared with continuous control, intermittent control is a more economical and effective approach when the system output is measured intermittently rather than continuously. On the other hand, an extreme case of intermittent control can be taken as impulsive control, in comparison with impulsive control, which is easier to implement in practical applications and process control since it has a nonzero control width, but impulsive control is activated only at some isolated instants. Owing to those merits, intermittent control has become a hot control method in the fields of chaotic systems and networks, and achieved many results, see, e.g., [34–52] and references therein.

So far, the Lyapunov function is the most common method to obtain stabilization criteria and synchronization criteria of chaotic systems and neural networks under intermittent control, but this method is somewhat conservative due to the construction of Lyapunov function, Sanchez and Perez matrix inequality, and techniques of differential inequality used. In [45–49], the authors analyzed the exponential stabilization problem of chaotic system and chaotic neural networks without delay and with constant delay. Unfortunately, some strong assumptions were made in these papers. For example, the control width is assumed to be half of control period [47], the control width has to be larger than the time delay [48], the relationship between non-control width and time delay is imposed [49]. In [50], the authors considered exponential stabilization and synchronization of neural networks with time-varying delays based on p-norm and ∞-norm, but the derivative of the time-varying delay has to be smaller than 1. These assumptions restrict the application scope for their results to some

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extent. Moreover, transcendental equations have to be solved which are computationally difficult in their results.

Time delays, especially time-varying delays, are unavoidably encountered in the signal transmission among the neurons, which will affect the stability of neural networks and may lead to some complex dynamic behaviors. Choosing an appropriate Lyapunov–Krasovskii functional that includes more useful state information and delay information is a more effective method to deal with this question than using Lyapunov function. However, based on the Lyapunov–Krasovskii functional method, for exponential stabilization of systems with time-varying delays via periodically intermittent control, few results are found in the literature. In [51,52], based on Lyapunov–Krasovskii functional and Jensen integral inequality, the authors discussed the exponential stabilization of neutral-type neural networks and chaotic systems with mixed time-varying via intermittent control respectively, but the obtained criteria are somewhat conservative. In [51], the criterion proposed is only applied to stabilize slow time-varying neural networks, and it is not be available to the unstable system, especially the chaotic system. Moreover, the criterion is expressed in the form of nonlinear matrix inequalities, which will increase computational complexity. In [52], the criteria proposed under periodically intermittent control cannot deal with the differentiable time-varying delays systems, and would not be feasible if the sizes of delays are ignored in the intermittent controller.

Motivated by the above analysis, in this paper, the problem of exponential stabilization for neural networks with time-varying delay is studied via periodically intermittent control. By employing the free-matrix-based integral inequality and some new analysis techniques, new delay-dependent sufficient conditions for exponential stabilization of neural networks are obtained in terms of linear matrix inequalities. A numerical simulation is provided to show the effectiveness and the benefits of the theoretical results.

This paper is organized as follows. In Section 2, model description and preliminaries are given. Some new criteria are obtained in Section 3 to ensure the exponential stabilization of neural networks. In Section 4, the effectiveness of the theoretical results is shown by a numerical example.

Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\|\cdot\|$ is the Euclidean norm of a vector; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; $P>0(<0, \le 0, \ge 0)$ means that the matrix is symmetric positive (negative, semi-negative, semi-positive) definite matrix; 'I' is an appropriately dimensioned identity matrix; $\lambda_{\min}(P)$ stands for the minimum eigenvalue of the matrix P; $Sym\{X\}=X+X^T$; $\sup_{x\in[a,b]} f(x)$ denotes the minimum value of upper bounds of the function $f(x)$ on the interval $[a,b]$; $*$ represents the symmetric block of a symmetric matrix.

2. Preliminaries

In this section, a class of neural networks with a time-varying delay is considered, and its model is represented as follows

$$\begin{cases} \dot{x}(t)=-Cx(t)+Af_1(x(t))+Bf_2(x(t-\tau(t)))+u(t), & t\ge 0, \\ x(t)=\varphi(t), \quad \forall t\in[-\tau,0], \end{cases} \tag{1}$$

where $x(t)=[x_1(\cdot)\ x_2(\cdot)\ \cdots\ x_n(\cdot)]^T\in\mathbb{R}^n$ is the neural state vector at time t, $f_i(\cdot)=[f_{i1}(\cdot)\ f_{i2}(\cdot)\ \cdots\ f_{in}(\cdot)]^T\in\mathbb{R}^n$ $(i=1,2)$ is the neural activation function which presents the nonlinear parameter perturbations, $u(t)$ is the control input vector; $C=diag\{c_1,c_2,\dots,c_n\}$ is a diagonal matrix with $c_i>0$, A, $B\in\mathbb{R}^{n\times n}$ are the connection weight matrices between neurons. The initial condition $\varphi(t)$ denotes a continuous vector-valued initial function on the interval $[-\tau,0]$.

Assumption (H_1)**.** The time delay, $\tau(t)$, is a time-varying differentiable function that satisfies

$$0\le\tau(t)\le\tau, \quad \dot{\tau}(t)\le\mu. \tag{2}$$

where τ and μ are real constants.

Assumption (H_2)**.** The nonlinear activation function $f_i(\cdot)$ $(i=1,2)$ with $f_i(0)=0$ satisfies the following condition, namely, there exist two positive diagonal matrices L_i such that

$$\|f_i(x)-f_i(y)\|^2\le(x-y)^TL_i(x-y). \tag{3}$$

for any $x,y\in\mathbb{R}^n$.

Remark 1. In fact, the above assumption implies activation that function $f_i(\cdot)$ satisfies the Lipschitz condition. For all $x,y\in\mathbb{R}^n$, the following inequalities hold

$$|f_{ij}(x)-f_{ij}(y)|\le\sqrt{\lambda_{L_i}}|x-y|, \quad (i=1,2; j=1,2,\dots,n).$$

where λ_{L_i} denotes the largest eigenvalue of L_i.

For system (1) with initial value, we consider an intermittent state feedback controller expressed as follows

$$u(t)=\begin{cases} Kx(t), & t\in[lT, lT+\delta), \\ 0, & t\in[lT+\delta,(l+1)T). \end{cases} \tag{4}$$

for any nonnegative integer l, where K is a constant control gain, T is the control period, $0<\delta\le T$, and δ is the so-called control width.

When the intermittent state-feedback control (4) is applied to (1), system (1) can be rewritten as follows

$$\begin{cases} \dot{x}(t)=-(C-K)x(t)+Af_1(x(t))+Bf_2(x(t-\tau(t))), & t\in[lT,lT+\delta), \\ \dot{x}(t)=-Cx(t)+Af_1(x(t))+Bf_2(x(t-\tau(t))), & t\in[lT+\delta,(l+1)T), \\ x(t)=\varphi(t), \quad \forall t\in[-\tau,0]. \end{cases} \tag{5}$$

Definition 1. System (1) is said to be exponentially stabilizable via intermittent state feedback control (4), if there exist $\alpha>0$ and $N>0$ such that the solution $x(t,\varphi)$ of system (5) satisfies

$$\|x(t,\varphi)\|\le Ne^{-\alpha t}\|\varphi\|. \quad \forall t\ge 0,$$

where $\|\varphi\|=\sup_{t\in[-\tau,0]}\|\varphi(t)\|$.

We introduce the following lemmas, which will be used in the proof of the main results.

Lemma 1. *(Free-matrix-based integral inequality [54]). Let $x:[\alpha,\beta]\to\mathbb{R}^n$ be a differentiable function. For symmetric matrices $R\in\mathbb{R}^{n\times n}$, X, $Z\in\mathbb{R}^{3n\times 3n}$, and any matrices $Y\in\mathbb{R}^{3n\times 3n}$, N_1, $N_2\in\mathbb{R}^{3n\times n}$ satisfying*

$$\begin{bmatrix} X & Y & N_1 \\ * & Z & N_2 \\ * & * & R \end{bmatrix}\ge 0$$

the following inequality holds:

$$-\int_\alpha^\beta \dot{x}^T(s)R\dot{x}(s)ds\le\varpi^T\hat{\Omega}\varpi \tag{6}$$

where

$$\hat{\Omega}=(\beta-\alpha)(X+\tfrac{1}{3}Z)+Sym\{N_1G_1+N_2G_2\},$$
$$G_1=\bar{e}_1-\bar{e}_2, \quad G_2=2\bar{e}_3-\bar{e}_1-\bar{e}_2,$$
$$\bar{e}_1=[I\ 0\ 0], \quad \bar{e}_2=[0\ I\ 0], \quad \bar{e}_3=[0\ 0\ I],$$
$$\varpi=\left[x^T(\beta)\ x^T(\alpha)\ \frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds\right]^T$$

Remark 2. As noted Remark 2 in [54], Lemma 1 includes Corollary 5 in [53], while the well-known Jensen inequality is a special case