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# Some novel approaches on state estimation of delayed neural networks<sup>\*</sup>



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#### ABSTRACT

This paper studies the issue of state estimation for a class of neural networks (NNs) with time-varying delay. A novel Lyapunov-Krasovskii functional (LKF) is constructed, where triple integral terms are used and a secondary delay-partition approach (SDPA) is employed. Compared with the existing delay-partition approaches, the proposed approach can exploit more information on the time-delay intervals. By taking full advantage of a modified Wirtinger's integral inequality (MWII), improved delay-dependent stability criteria are derived, which guarantee the existence of desired state estimator for delayed neural networks (DNNs). A better estimator gain matrix is obtained in terms of the solution of linear matrix inequalities (LMIs). In addition, a new activation function dividing method is developed by bringing in some adjustable parameters. Three numerical examples with simulations are presented to demonstrate the effectiveness and merits of the proposed methods.

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#### 1. Introduction

Neural networks (NNs) have attracted a great deal of attention in recent years due to their important applications in many applied fields such as pattern recognition, signal processing, adaptive control, and combinatorial optimization [6,11,13,20,40,43,46]. Their dynamical behaviors, such as stability, attraction, and oscillation, have become hot research topics studied by numerous researchers around the globe. But in practical applications, stability is a key property needed in the design of NNs.

In the implementation of NNs, it is often inevitable to introduce time delay in the signals transmitted among neurons [16,47,48,52]. Hence it is practical to study delayed neural networks (DNNs). DNNs have achieved high recognition for speech data and have the ability to tolerate the time lag caused by variation in the phoneme extraction position (time-shifting invariance) [18]. It is also used to capture the temporal relationship between predictions on continuous instances of facial expression video recording [27]. However, time delay may cause instability, oscillation, or poor performance of NNs. Therefore, the stability problem of DNNs has been recognized as an important issue. Numerous important and interesting research

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results have been developed in [3–5,21,38,41]. Based on a linear matrix inequality (LMI) approach, the global asymptotical stability analysis for a class of stochastic NNs with mixed time delays was investigated in [37]. By making full use of the information of neuron activation function, a new exponential criterion was proposed in [39]. The authors in [42] discussed the problem for robust exponential stability for a class of uncertain stochastic NNs with multiple delays by using free-weighing matrix method. The issue of dissipativity analysis of memristor-based complex-valued NNs with time-varying delays was studied in [26].

Although DNNs have been used as viable network models the neuron states are seldom fully available in the network outputs, especially in relatively large-scale NNs. Consequently, neuron state estimation becomes an important research topic in many practical applications. Paper [36] studied the state estimation problem for NNs with time-varying delay through available output measurements. Since then, the investigation of the state estimation in the case of DNNs has gained rapid development. A number of available measurements and kinds of valid methods have been proposed in [8–10,12,45,51]. For instance, a delay-dependent criterion was developed to estimate the neuron states through available output measurements and the free-weighting matrix approach in [8]. However, the obtained stability criterion in [8] was presented in terms of a matrix inequality, rather than an LMI, which corresponds to a nonlinear programming problem and is generally very difficult to deal with. Paper [9] considered the robust state estimation problem for a class of uncertain DNNs based on a bounding technique. Different from the assumptions in [8], the boundedness of the time-varying delay was only required by defining a new LKF in [10]. New delay-dependent stability criteria for DNNs were obtained by using a delay-partition approach (DPA) in [12,45,50,51]. The advantage of this approach is that less conservative stability criteria can be achieved without introducing any slack variables.

In order to reduce ulteriorly the conservatism of stability criteria for DNNs, the reciprocally convex optimization technique [31,32] was fully applied to the DPA in [7,42]. Alternatively, inspired by the above division, the activation function dividing approach was proposed to study the problem of delay-dependent stability criteria for DNNs in [19,24]. By introducing a tuning parameter, this approach in [19] was modified for investigating an extended dissipative analysis of DNNs in [24]. Besides, other effective methods were utilized, such as a suitable LKF including double and triple integral terms [23], zero equalities and reciprocally convex approach [25], free-weighting matrix technique [17,33], a new convex combination technique [1,49] and a decoupling technique [14]. However, these results appear to have some common shortcomings. On the one side, the relationship between time-varying delay and each subinterval is not taken sufficiently into account. On the other side, some useful integral terms and more information of neuron activation functions are not well utilized, see [9,10,12,51], which may obtain a smaller time-delay upper bound to a certain extent.

Motivated by the proceeding discussion, we investigate, in this paper, the state estimation problem of NNs with timevarying delay and establish some less conservative results by using some more effective methods and novel approaches. The main contribution of this paper lies in the following three aspects. In the first place, different from the existing methods in [19,24], we propose a general bounding partitioning method of activation function by introducing *n* variable parameters, which plays a key role in obtaining less conservative stability conditions. Moreover, the methods in references [19,24] can be considered as special cases of the proposed approaches in this paper. In the second place, in order to obtain new stability results, a more general SDPA is proposed for constructing an augmented LKF, which is not used in [19,24]. The total interval of the time-varying delay is divided into two alterable subintervals, and then each subinterval is further divided into two variable parts. Compared with the approaches in [7,12,34,45,50,51], the proposed approach is able to take full account of the relationship between time-varying delay and each subinterval. In the third place, by using a MWII, which is less conservative than the celebrated Jensen's inequality used in [7,12,19,24,34,45,50,51], the state desired estimator can be achieved by solving a set of LMIs. Finally, three examples are given to demonstrate the effectiveness and advantages of the developed results.

**Notations:** Notations used in this paper are fairly standard: Let  $\mathbb{R}$  be the real line, I the identity matrix of appropriate dimensions,  $\mathbf{A}^T$  the matrix transposition of the matrix  $\mathbf{A}$ ,  $\mathbf{B}^{-1}$  the inverse matrix of the matrix  $\mathbf{B}$ . By  $\mathbf{X} > 0$  (respectively  $\mathbf{X} \geq 0$ ), for  $\mathbf{X} \in \mathbb{R}^{n \times n}$ , we mean that the matrix  $\mathbf{X}$  is real symmetric positive definite (respectively, positive semi-definite);  $diag\{r_1, \dots, r_n\}$  diagonal matrix with diagonal elements  $r_i, i = 1, \dots, n$ , the symbol \* represents the elements below the main diagonal of a symmetric matrix,  $\vec{\mathbf{S}}$  is defined as  $\vec{\mathbf{S}} = \mathbf{S} + \mathbf{S}^T$ .

#### 2. Preliminaries

Consider the following NNs with time-varying delay:

$$\dot{\mathbf{x}}(t) = -\mathbf{W}_0 \mathbf{x}(t) + \mathbf{W}_1 \mathbf{g}(\mathbf{x}(t)) + \mathbf{W}_2 \mathbf{g}(\mathbf{x}(t - d(t))) + \mathbf{J}_2$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) + \tilde{\mathbf{g}}(t, \mathbf{x}(t)), \tag{1}$$

where  $\mathbf{x}(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^n$  is the neuron state vector,  $\mathbf{g}(\mathbf{x}(t)) = [g_1(x_1(t)), \dots, g_n(x_n(t))]^T \in \mathbb{R}^n$  is the neuron activation function, and  $\mathbf{J} = [J_1, \dots, J_n]^T \in \mathbb{R}^n$  is an external constant input vector.  $\mathbf{W}_0 = diag\{w_{01}, \dots, w_{0n}\} > 0$ ,  $\mathbf{W}_1 \in \mathbb{R}^{n \times n}$  is the interconnection weight matrix,  $\mathbf{W}_2 \in \mathbb{R}^{n \times n}$  is the delayed interconnection weight matrix, and  $\mathbf{C} \in \mathbb{R}^{m \times n}$  is the output weight matrix.  $\mathbf{y}(t) \in \mathbb{R}^m$  is the measurement output of the networks,  $\tilde{\mathbf{g}}(t, \mathbf{x}(t)) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^m$  is the neuron-dependent nonlinear disturbance on the network outputs, and d(t) is the time-varying discrete delay.

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