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Novel basic operational laws for linguistic terms, hesitant fuzzy linguistic term sets and probabilistic linguistic term sets

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ABSTRACT

In the process of decision making, people sometimes may feel more comfortable to express their preferences by linguistic terms instead of the quantitative form. However, as the basic premise of operations, the existing operational laws of linguistic terms and the extended linguistic term sets are very unreasonable. In order to overcome this issue, in this paper, we redefine some more logical operational laws for linguistic terms, hesitant fuzzy linguistic elements (HFLEs) and probabilistic linguistic term sets (PLTSs) based on two equivalent transformation functions. These novel operational laws can not only avoid the operation values exceeding the bounds of LTSs, but also keep the operation results more reasonable in decision making with linguistic information. Furthermore, the operational laws can keep the probability information complete when computing with PLTSs. Additionally, lots of properties of the operational laws are discussed, and some three-dimensional figures are drawn to show the regions of different operational laws of linguistic terms more vividly.

1. Introduction

Decision making is a common activity in our daily life. Over the past decades, a lot of decision making techniques have been developed, including the TOPSIS methods [2,5,21,23], the VIKOR method [12], the LINMAP method [16] and granular computing [1,3,4,14,17-19,22-24,26,31,37,38], etc. In the process of decision making, people sometimes may feel more comfortable to express their preferences by linguistic terms instead of the quantitative form. Therefore, the fuzzy linguistic approach [39] has received lots of scholars' attention, and it is an effective way to model linguistic information. In this approach, an important step is to transform the linguistic information into a machine manipulative format, in which the computation can be carried out [12]. Therefore, Xu [34] proposed a subscript-symmetric additive linguistic term set (LTS), but it is discrete and sometimes inconvenient for calculation and analysis. In order to preserve all linguistic information, he further extended the discrete LTS to a continuous LTS (or called virtual LTS), and based on which, Liao et al. [9] established the mapping between virtual linguistic terms and their corresponding semantics as shown in Fig. 1.

Later on, a series of extended LTSs have been put forward, such as hesitant fuzzy linguistic term set (HFLTS) [2,5-12,15,29,30,33,34,40-42], linguistic hesitant fuzzy set (LHFS) [16,20], probabilistic linguistic term set (PLTS) [21], etc. The HFLTS, combining the LTS and the hesitant fuzzy set (HFS), was introduced by Rodríguez et al. [25]. It is a more reasonable information expression form, which can be used to describe people's subjective cognitions. Liao et al. [11] redefined the concept of HFLTS in term of mathematical representation, and the elements of a HFLTS were called hesitant fuzzy

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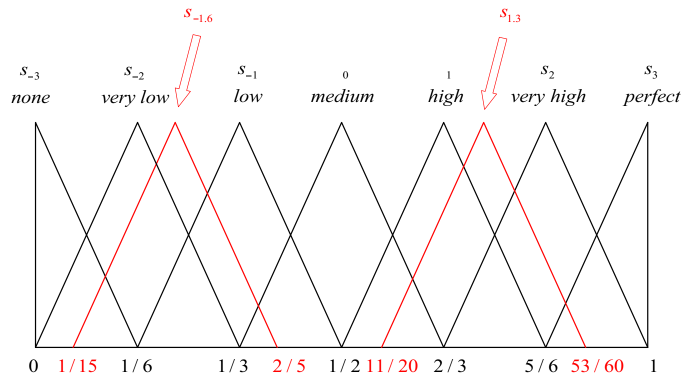

Fig. 1. Semantics of virtual linguistic terms.

linguistic elements (HFLEs). In recent years, a lot of scholars have studied HFLTSs from different angles, and developed the hesitant fuzzy linguistic information aggregation theory [7,30,33,40], the hesitant fuzzy linguistic measurement theory [6,8,9-11,28], the hesitant fuzzy linguistic preference relation theory [11,15,41,42], and the hesitant fuzzy linguistic decision making methodologies [2,5,12,28,29], etc.

However, due to the lack of considering the weight information in most of the current studies about HFLTSs, we always give tacit consent to all the linguistic terms having the same importance or weight, but it rarely happens in reality. In fact, people may prefer some of the possible linguistic terms so that they may have different importance degrees. Therefore, Pang et al. [21] introduced PLTS to extend HFLTS through adding probabilities without loss of any original linguistic information. Additionally, they developed an extended TOPSIS method and an aggregation-based method respectively for multi-attribute group decision making (MAGDM) with probabilistic linguistic information.

In general, we need to make some operations when dealing with all kinds of linguistic information in practical decision making problems. However, the existing operational laws of linguistic terms have some shortcomings as follows:

(1) Suppose that $S=\{s_t|t=-3,\dots,-1,0,1,\dots,3\}$ is a LTS, s_2 and s_3 are two linguistic terms, and let $\lambda=2$, then by the basic operational laws of linguistic terms given by Xu [35], we obtain $s_2\oplus s_3=s_5$ and $\lambda s_2=s_4$. Obviously, both these two results exceed the upper bound s_3. Considering that the operations of HFLEs are based on the operational laws of linguistic terms [35], they also have this drawback.

(2) Let S be the LTS as defined above, and let $L_1(p)=\{s_1(0.2),s_2(0.3),s_3(0.5)\}$ and $L_2(p)=\{s_2(0.2),s_3(0.8)\}$ be two PLTSs, then by using the operational laws of PLTSs given by Pang et al. [21], we obtain $L_1(p)\oplus L_2(p)=\{0.2s_1\oplus 0s_2, 0.3s_2\oplus 0.2s_2, 0.5s_3\oplus 0.8s_3\}=\{s_{0.2},s_1,s_{3.9}\}$. Clearly, the result $s_{3.9}$ not only exceeds the bound $[s_{-3},s_3]$, but also loses the corresponding probability information.

In order to avoid these issues, we need to define some novel operational laws for linguistic terms, HFLEs and PLTSs. Gou et al. [7] proposed two equivalent transformation functions g and g^{-1}, which can achieve the equivalent transformations between the HFLEs and the HFEs. Based on g and g^{-1}, in this paper we define some novel operational laws of linguistic terms, HFLEs and PLTSs, including addition, multiplication, power, subtraction, division and supplement. These operational laws not only can avoid the operation results exceeding the bounds of LTSs, but also can keep the probability information complete after operations. Fig. 2 describes the operation process of the novel operational laws clearly.

The remainder of this paper is organized as follows: In Section 2, we review some concepts of LTSs, HFLTSs and PLTSs, and introduce two equivalent transformation functions g and g^{-1}. In Section 3, we define some novel operational laws for linguistic terms and discuss their properties. We also draw some three-dimensional figures to show the regions of different operational laws vividly. Additionally, some novel operational laws of HFLTSs are defined and their properties are also discussed in Section 4. Section 5 defines the operational laws of PLTSs considering the factors of linguistic terms and probability information. In Section 6, we use a practical example to show the novel operational laws of HFLEs and PLTSs for dealing with a multiple criteria decision making (MCDM) problem. We also summarize the advantages of these operational laws by comparing them with the existing ones. Finally, we draw some conclusions and point out the future research directions of the linguistic terms, HFLEs and PLTSs in Section 7.

2. Linguistic term set, hesitant fuzzy set and some extended forms

In this section, we mainly recall some basic concepts and properties of LTSs, HFSs, HFLTSs and PLTSs: