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A new consensus ranking approach for correlated ordinal information based on Mahalanobis distance

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ABSTRACT

We investigate from a global point of view the existence of cohesiveness among experts' opinions. We address this general issue from three basic essentials: the management of experts' opinions when they are expressed by ordinal information; the measurement of the degree of dissensus among such opinions; and the achievement of a group solution that conveys the minimum dissensus to the experts' group.

Accordingly, we propose and characterize a new procedure to codify ordinal information. We also define a new measurement of the degree of dissensus among individual preferences based on the Mahalanobis distance. It is especially designed for the case of possibly correlated alternatives. Finally, we investigate a procedure to obtain a social consensus solution that also includes the possibility of alternatives that are correlated. In addition, we examine the main traits of the dissensus measurement as well as the social solution proposed. The operational character and intuitive interpretation of our approaches are illustrated by an explanatory example.

1. Introduction

A considerable amount of literature has contributed to the research issue of obtaining consensus in group decision making problems. This issue is an active subject in several areas such as Social Choice Theory and Decision Making Theory. From the Social Choice perspective several contributions can be emphasized, e.g., [2,4,9,25,26,49], among others. From the Decision Making Theory, it has been successfully tackled by a great amount of contributions, e.g., [28,32,35,36,58], among others. Besides these main areas, there are some other methodologies that proposed different definitions of the consensus concept. It is worth mentioning the work of González Jaime et al. [38] and López Molina, De Baets and Bustince [47].

Any group decision making problem focused on obtaining consensus involves at least three key pillars. The first one is the way in which experts give their opinions on a set of alternatives and how such an information is managed. Once the opinions of the agents have been gathered it seems natural to measure how much cohesiveness these opinions generate. Thus, the second pillar is to establish a mechanism able to provide such measurements. Apart from determining the degree

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of consensus among experts the main aim of a group decision making problem is to determine a solution. The better solution the greater agreement this solution generates among experts. Consequently, supplying a method to achieve a group consensus solution is the third pillar.

We now briefly review the previous literature related to each basic essentials.

Information formats. Generally speaking, experts can express their opinions by means of ordinal or cardinal information, the former being more extensively used in the research issue addressed in this work. Nonetheless, contributions dealing with cardinal information include the approaches proposed by Herrera-Viedma, Herrera and Chiclana [36], González-Pachón and Romero [30] and González-Arteaga, Alcantud and de Andrés Calle [26]. The representation of ordinal information has been a subject of study for over two centuries for linear orders (see e.g., [3,9]), weak orders(see e.g., [4,16,24]) and fuzzy preferences (see e.g., [12,22,51,56], among others).

Regardless of the experts' information format, it is necessary to manipulate it in order to make suitable computations. In the literature several procedures to codify linear and complete preorders into numerical values can be found (see [7,8,14,25], among others), Borda [8] being the first author to manage ordinal preferences in such way.

Consensus measurement. This topic was initiated by Bosch [9] from the Social Choice perspective. In this vein McMorris and Powers [49] characterized consensus rules defined on hierarchies, while García-Lapresta and Pérez-Román [25] introduced a class of consensus measures based on distances. Subsequently, Alcalde-Unzu and Vorsatz [2,3] proposed and characterized a family of linear and additive consensus measures based on measuring similarity among preferences. From another point of view, Alcantud, de Andrés Calle and Cascón [5,6] introduced the analysis when opinions are dichotomous.

The use of distance and similarity functions has provided interesting insights about cohesiveness measurement. We highlight the role of the Kemeny, Mannhattan, Jacard, Dice and Cosine distance functions (see e.g., [5,13,15,25]). Moreover, it is also possible to apply some association measures to that purpose (see e.g., [14,21,27,33,42,55]).

Group consensus solution. Finding the best option or solution from alternatives is the main aim in group decision making problems. Recently, various approaches have been developed to solve this problem from a variety of science areas: *Operational Research* (see e.g., [17,20]), *Statistical Analysis* (see e.g., [1,23,45]), *Fuzzy Theory* (see e.g., [18,46,59]), and *Computational Analysis* (see e.g., [37,60]).

Traditionally, the achievement of a global solution has been considered as an aggregation problem of experts' opinions in order to obtain a social solution. Different methods have been proposed and analyzed for aggregating agents' opinions (preferences in the case of ordinal information) into a social solution. Borda [8] first examined this problem in a voting context and Kendall [41] subsequently revised Borda's method in a statistical framework.

Other authors also proposed alternative distance-based aggregation rules e.g., Eckert and Klamler [19], Klamler [43,44], Meskanen and Nurmi [50], Ratliff [52,53], and Saari and Merlin [54], even though Kemeny's rule [39] could be considered as a landmark in aggregation procedures based on distances. Following Kemeny's rule, Cook and Seiford [14] established an equivalence between the Borda-Kendall method [40] and their approach. González-Pachón and Romero [28] developed a general framework for distance-based consensus models under the assumption of a generic l_p metric. These authors have recently designed socially optimal decisions in a consensus scenario [31].

Once we have reviewed the related literature we now summarize the main contributions of this paper.

- We focus on group decision making problems where agents or experts provide their opinions on a set of alternatives by complete preorders. In this regard, we propose a new codification procedure to transform the original opinions/preferences of agents into numerical vectors in order to manage them. For the purpose of better understanding this process we investigate exactly which vectors are realizations by a *canonical codification procedure* of generic complete preorders. The characterization of the new codification procedure is a key point because it ensures consistency of our approach and its use in any methodology.
- In order to measure the degree of cohesiveness among agents' preferences, we design an indicator of dissensus for a finite collection of complete preorders on a finite set of alternatives based on the Mahalonobis distance, which is dependent on a positive definite matrix (the parameter) that captures the importance and possible cross-relations of each alternative, namely, the *Mahalanobis dissensus measures*. Any such indicator ranks the profiles of complete preorders (in the form of codified matrices) according to their inherent cohesiveness. The strength of our measurement unlike other aforementioned approaches based on distances is the inclusion of the relationships among alternatives. Then, the new measure incorporates relevant information that in other way is ignored. Moreover, we investigate the main characteristics of the novel measure and prove that a partial order can be naturally induced on the parametric class of all *Mahalanobis dissensus measures*.
- Then we exploit these measures in order to propose a consensus solution especially designed for profiles of preferences on possibly correlated alternatives and to overcome the drawbacks of the aforementioned distance-based methodologies. That solution aggregates individual opinions into a social preference on the alternatives by minimizing dissensus with respect to the original profile of preferences. In order to facilitate the computation of such compromise solution we prove that the problem is equivalent to minimizing the Mahalanobis distance to a single average vector. Whatever the statement of the minimization problem, the objective function is restricted to feasible codified vectors, which emphasizes