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MNiBLoS: A SMT-based solver for continuous t-norm based logics and some of their modal expansions

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A B S T R A C T

In the literature, little attention has been paid to the development of solvers for systems of mathematical fuzzy logic, and in particular, there are few works concerned with infinitely-valued logics. In this paper it is presented *mNiBLoS (a modal Nice BL-Logics Solver)*: a modular SMT-based solver complete with respect to a wide family of continuous t-norm based fuzzy modal logics (both with finite and infinite universes), restricting the modal structures to the finite ones. At the propositional level, the solver works with some of the best known infinitely-valued fuzzy logics (including BL, Łukasiewicz, Gödel and product logics), and with all the continuous t-norm based logics that can be finitely expressed in terms of the previous ones; concerning the modal expansion, mNiBLoS imposes no boundary on the cardinality of the modal structures considered. The solver allows to test 1-satisfiability of equations, tautologicity and logical consequence problems. The logical language supported extends the usual one of fuzzy modal logics with rational constants and the Monteiro-Baaz Δ operator. The code of mNiBLoS is of free distribution and can be found in the web page of the author.

1. Introduction

Mathematical Fuzzy Logic (MFL) is a sub discipline of Mathematical Logic that studies a certain family of formal logical systems whose algebraic semantics involve some notion of truth degree. These truth degrees have motivations coming from different fields, like philosophy, fuzzy set theory and many-valued logics. Along the XXth century, and particularly from the 90s, MFL have been widely studied, and there is a large number of studies on the applications of this framework to model knowledge and reasoning with incomplete and uncertain information. However, in the literature, with a few exceptions mainly for finitely valued Łukasiewicz logics [34–36], little attention has been paid to the development of efficient solvers for systems of mathematical fuzzy logic, even though there is an important number of studies on complexity and proof theory for them [3,24,26,27,33]. This is a problem that limits the use of fuzzy logics in real applications, while the number of theoretical proposals relying on these logics has rapidly grown. Moreover, the relative poor knowledge of more complex t-norm based logics outside the fuzzy logic community, and in particular, in more applied areas, limits the use of these logics in situations where ad-hoc logics (i.e., logics where the behavior of the connectives is defined accordingly to the user's necessities) could be very interesting.

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In [5] it is proposed a new approach for implementing a theorem prover (i.e., that checks validity of a given formula) for Łukasiewicz, Gödel and product fuzzy logics using Satisfiability Modulo Theories (SMT). The idea of using SMT solvers in order to solve this problem is new and allows to treat infinitely-valued logics without relying on reductions to finite universes, and so being able to solve the validity problem over the previous logics. The results are very interesting in particular for Łukasiewiczand Gödel logics because the execution times turn out to be optimistically efficient. In the case of product logic, while the importance of this new solver is clear since no previous automatic tool for product logic existed before, the results are however not very satisfactory: the execution times are quite long, mainly due to the fact that the implementation of this logic uses theories of the SMT-solver with non-linear arithmetic. Inspired by the previous work, in [40] it is presented a solver for the whole family of logics based on a (finitely representable) continuous t-norm and BL, and the particular case of the product logic is strongly enhanced relying on theoretical results from [23]. However, if truth constants are added to the language, their behavior (for what concerns the product logic) following this new approach is completely uncontrolled. Moreover, it is not studied how to include this new treatment of the product logic when considering arbitrary continuous t-norms that included some product t-norm in their representation as ordinal sums. Thus, the new approach is limited to product logic only.

On the other hand, modal expansions of MFL, which allow to reason over *qualified* concepts (like always, sometimes, etc.), have also been intensively studied in the latter years, partially motivated by the rich expressive power of these logics and their possible applications. Concerning the development of applications oriented to automatically reason over these kind of logics, the main works lie within the framework of the so-called Fuzzy Description Logics (FDL), multi-modal logics oriented to represent ontologies of different areas (see for instance [10,38,39]). In relation to the present paper, we can cite two works presenting a reasoner for some FDLs.

In [11] a solver (fuzzyDL) that checks satisfiability over Łukasiewicz and Zadeh logics is presented. fuzzyDL is correct and complete for Zadeh logics, while is correct and complete for Łukasiewicz logic if the TBox is unfoldable [13]. Furthermore, in [14] it is shown that fuzzyDL can deal with the satisfiability and entailment problems in a very efficient way over the implemented logics. However, let us remark that fuzzyDL is not oriented to work with other logics like for instance Hajek's BL or product logic. On the other hand, some notes towards the implementation of a solver that works with the problem of positive satisfiability for product description logic can be found in [4]. The approach followed to solve reason at a propositional level over the product standard algebra is the same one followed in [40]. On the other hand, concerning the expansion to FDL, it relies in the possibility of reducing a problem of positive satisfiability to the satisfiability of a quantifier free boolean formula, which is, however, not likely to work in more general cases (like 1-satisfiability or validity). Moreover, the resulting formula has non-linear real arithmetic properties, making it quite challenging, as we already commented above, from an efficiency point of view.

Our objective within this paper is expanding the solver presented in [40] and solve several problems left open in the previous work and in [5]. We present a solver (mNiBLoS) with a quite simple interface, a rich expressive power and a reasonable level of efficiency, that hopefully can help a non-specialized community to work with several fuzzy modal logics through a quite short process of intuitive learning. Even if the software presented is command-line only, interested users needs not to be more than slightly familiar with continuous t-norms and logical languages in order to be able to use it: only the decomposition of the t-norm, the desired task and the formulas/equations associated to the desired task are asked to the user. We offer the possibility of working with a large set (countable infinitely many) of infinitely valued logics, including Hajek's Basic Logic and logics based on finite ordinal sums of Gödel, Łukasiewiczand product t-norms. mNiBLoS also supports a rich language, including rational truth constants, Δ and the usual modal operators $\Box$ and $\Diamond$. We have implemented the basic operations that seem to be of most interest concerning the use of a fuzzy modal logic: validity, logical deduction and 1-satisfiability (united to the generation of a solution when possible) of sets of formulas or equations. Focusing on the efficiency of the solver, we have proven that reasoning over a logic based on a continuous t-norm that includes a product t-norm can be equally done over the same structure but whose product components are moved into the negative cone of the real numbers, by means of a translation that allows to keep working with rational truth constants. Moreover, we give a constructive proof of the fact that the operations we are concerned with are decidable over finite Kripke structures, in a similar fashion as it was done in [28] for Łukasiewicz FDL. For doing this, we present an algorithm that builds the minimum (finite) structure that is necessary to check in order to determine, for a finite set of formulas/equations, the answer for the theoremhood, logical deduction and satisfiability problems.

Structure of the paper. Section 2 introduces the propositional logics considered and some of their characteristics, the modal expansions we are going to implement and gives a brief introduction to SMT. Section 3 presents the theoretical results developed for the design of mNiBLoS: the approach to product logic based on linear arithmetic, the treatment of BLand the study of the expansions with truth constants and modal operators. Section 4 comments on the design decisions of the solver and explains some details of its implementation, and shows several empirical studies run on mNiBLoS. We conclude the paper with Section 5, by outlining the conclusions after this research and some possible future work.

2. Preliminaries

With the objective of being as self-contained as possible, we first describe in this section the logical background necessary to present the theoretical results of Section 4. With this in mind, we mainly focus on semantic aspects of the logics,