Neurocomputing 207 (2016) 528–538

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Global exponential stability of periodic solution for delayed complex-valued neural networks with impulses ☆

Dong Xie [a,b,*], Yueping Jiang [a]

[a] College of Mathematics and Econometrics, Hunan University, Changsha 410082, China
[b] Department of Electronic and Information Engineering, Bozhou University, Bozhou 236800, China

ARTICLE INFO

Article history:
Received 13 December 2015
Received in revised form
14 March 2016
Accepted 26 April 2016
Communicated by Y. Liu
Available online 20 May 2016

Keywords:
Complex-valued neural networks
Periodic solution
Exponential stability
Impulses
Delays

ABSTRACT

In this paper, a class of delayed complex-valued neural networks with impulses is investigated. By using Mawhin's continuation theorem of coincidence degree theory, a series of useful criteria on existence of periodic solution are established for the complex-valued neural networks. By constructing appropriate Lyapunov–Krasovskii functional, some sufficient conditions are derived for the global exponential stability of periodic solutions to the complex-valued neural networks. Finally, several examples with numerical simulations are given to highlight the effectiveness of our theoretical results via standard numerical software.

© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Complex-valued neural networks (CVNNs for short) are the networks that deal with information in the complex plane, that is, their state vectors, connection weights and activation functions are complex-valued. CVNNs become a fast emerging field of research in both theoretical and application points of view due to their widespread applications in various fields, such as optoelectronics, filtering, imaging, speech synthesis, computer vision, remote sensing, quantum devices, spatio-temporal analysis of physiological neural devices and systems, and artificial neural information processing (see [1–8] and references therein), in recently several years, more and more researchers focus on their attentions and interests to CVNNs, and there have been some good results [3–25]. In some sense, CVNNs can be regarded as the extension of real-valued neural networks (RVNNs for short). However, CVNNs have the potential to solve problems that cannot be solved by their real-valued counterparts. For example [9], the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons. Compared with RVNNs, it is more difficult to study the dynamic behaviors of CVNNs, CVNNs are quite different and have more complicated properties than real-valued ones. Thus, it is important and necessary to study the dynamic behaviors of CVNNs. Recently, the dynamic behavior analysis of CVNNs has been widely studied and there are a large number of relevant publications (see [5–9,11–25] and references therein). Thus it is important to investigate CVNNs deeply in order to explore new capabilities and higher performance both theoretically and practically. There have been various approaches for analyzing the dynamic behaviors of CVNNs, such as the scalar Lyapunov function method [5], the energy function method [34], the LMI method [35], and the synthesis method [7], etc.

It is well known that the properties of periodic solutions are of great interest, which have been successfully applied in many neural network systems, such as many biological and cognitive activities (for example heartbeat, respiration, mastication, locomotion, and memorization) require repetition. Periodic oscillations in recurrent neural networks have found many applications, such as associative memories, pattern recognition, machine learning, and robot motion control. The research of neural networks with delays involves not only the stability analysis of equilibrium points but also that of periodic solutions. In particular, an equilibrium point can be considered as a special case of periodic solution for neural networks with arbitrary period. Therefore, the stability analysis of periodic solutions for neural networks are

☆The work was supported by the NSFC (Grant Nos. 11201134, 11371126), and PhD Students Innovation Foundation of Hunan Province (CX2015B089), the high school outstanding young support plan of Anhui Province, the natural science research project of Anhui Province (KJ2015A347) and the college teaching quality and teaching reform project of Anhui Province (2015gxk089).

* Corresponding author at: College of Mathematics and Econometrics, Hunan University, Changsha 410082, China.
E-mail addresses: dxie@hnu.edu.cn (D. Xie), ypjiang@hnu.edu.cn (Y. Jiang).

http://dx.doi.org/10.1016/j.neucom.2016.04.054
0925-2312/© 2016 Elsevier B.V. All rights reserved.

more general and interesting than that of equilibrium points. Recently, in CVNNs, there have been a lot of literatures for the stabilization of equilibrium points, see [5,11,12,14–24]. In RVNNs, there have been some good results for periodic solutions, see [26–33]. However, there are very few literatures for periodicity of CVNNs with time delays, see [25]. In [25], a discrete-time complex-valued recurrent neural network model is analysed, the sufficient conditions on its global exponential periodicity and exponential stability are obtained. However, to the best of our knowledge, there are no studies been reported on the problem of existence and exponential stability of periodic solution of delayed CVNNs with impulses until now.

As we know, time delays are commonly encountered, some experiments showed that time delays can affect the stability of neural networks and lead to some other dynamical behaviors such as oscillation, divergence, bifurcation, chaos, instability and so on. On the other hand, impulsive neural networks is neither purely continuous nor purely discrete, which can display a combination of characteristics of both continuous and discrete systems. The dynamic behaviors of such systems are much more complicated, compared with the behavior of the system without impulse effects. The impulses of the system can affect the dynamical behaviors of the system, same as time delays effect. Most of the results have focused on the stability analysis of equilibrium points of delayed CVNNs without impulsive effects, see [15–25]. Recently, increasing attention has been focused on stability analysis of impulsive real-valued neural networks with time delays [26–30]. However, there have been very few contexts for CVNNs with delays and impulses, for example [12], in [12], the analysis of complete stability of complex-valued neural networks with time delays and impulses is studied, some sufficient conditions for the existence and complete stability of CVNNs with time delays and impulses are derived in form of complex-valued linear matrix inequality (LMIs for short) by constructing Lyapunov–Krasovskii functional.

Motivated by the above discussions, the objective of this paper is to study existence and exponential stability of periodic solution of CVNNs with time delays and impulsive effects. Using Mawhin's continuation theorem of coincidence degree and by constructing appropriate Lyapunov–Krasovskii functional, some sufficient conditions for existence and global exponential stability of periodic solution of complex-valued neural networks with delays and impulses are derived. In these sufficient conditions, the activation functions only need to satisfy the boundedness and the Lipschitz condition. It removes the restrictions on the existence, continuity, and boundedness of the partial derivatives of the activation functions about their real and imaginary parts, the conditions of our paper are weaker than [5,23]. Moreover, Our method of Mawhin's continuation theorem of coincidence degree have never used in CVNNs. Finally, several numerical examples are given to establish the effectiveness of our theoretical results via standard numerical software.

The main contribution of this paper is organized as follows: in the next section, impulsive CVNNs model, hypotheses, some preliminary definitions, Lemmas and notations are presented. Section 3 addresses the existence of periodic solution of the CVNNs by using the continuation theorem of coincidence degree theory proposed in [36]. In Section 4, by constructing the appropriate Lyapunov–Krasovskii function we derive several sufficient conditions ensuring that the periodic solution of the CVNNs is global exponential stability. Then, several illustrative examples are given to show the effectiveness of the obtained results in Section 5.

2. Models and preliminaries

To begin with, we would like to introduce the following notations. Let T represent the matrix transposition. i denotes the imaginary unite, that is, $i=\sqrt{-1}$. $\mathbb{Z}^+$ denote the set of positive integers. $z(t)$ represents the complex-valued function, that is, $z(t)=x(t)+iy(t)$, where $x(t),y(t)\in\mathbb{R}^n$, $\mathbb{R}^n$ and $\mathbb{C}^n$ denote the n-dimensional real spaces and complex vector spaces respectively.

2.1. Models

In this paper, we consider the following impulsive complex-valued neural networks (ICVNNs) with time-varying delays described by:

$$\begin{aligned}\frac{dz_p(t)}{dt} &= -d_p(t)z_p(t)+\sum_{q=1}^{n} a_{pq}(t)f_q(z_q(t))\\ &\quad+\sum_{q=1}^{n} b_{pq}(t)f_q(z_q(t-\tau_{pq}(t)))+H_p(t), \quad t\neq t_k,\\ \Delta z_p(t_k) &= z_p(t_k^+)-z_p(t_k^-)=z_p(t_k^+)-z_p(t_k)=J_{pk}, \quad t=t_k, \quad k\in\mathbb{Z}^+,\end{aligned} \tag{2.1}$$

where $z_p(t)=x_p(t)+iy_p(t)$ denotes the complex-valued state vector associated with the p-th neuron, $p,q=1,2,\ldots,n$, n is the number of neurons. For convenience, $z_p(t),x_p(t)$ and $y_p(t)$ are denoted as z_p, x_p and y_p, respectively. This model describes the continuous evolution process of the neural networks. $d_p(t)\in\mathbb{R}$ is the self-feedback connection weight, $a_{pq}(t),b_{pq}(t)$ are complex-valued connection weight matrices without and with time delays respectively. $f_q(z_q):\mathbb{C}\to\mathbb{C}$ is the activation function of the neurons. $H_p(t)\in\mathbb{C}$ is the external input vector. $\tau_{pq}(t)\geq 0$ correspond to the transmission delays. $\Delta z_p(t_k)=z_p(t_k^+)-z_p(t_k^-)$ are the impulses at moments t_k and $0<t_1<t_2<\cdots$ is strictly increasing sequence such that $\lim_{k\to\infty}t_k=+\infty$, $\{t_k\}$ are called points of jump.

The model (2.1) can be rewritten in vector form as follows,

$$\begin{aligned}\frac{dz(t)}{dt} &= -D(t)z(t)+A(t)f(z(t))+B(t)f(z(t-\tau(t)))+H(t), \quad t\neq t_k,\\ \Delta z(t_k) &= z(t_k^+)-z(t_k^-)=z(t_k^+)-z(t_k)=J_k, \quad t=t_k, \quad k\in\mathbb{Z}^+,\end{aligned} \tag{2.1'}$$

where $z(t)=(z_1(t),z_2(t),\ldots,z_n(t))^T\in\mathbb{C}^n$ is the state vector, $D(t)=\mathrm{diag}(d_1(t),d_2(t),\ldots,d_n(t))\in\mathbb{R}^n$ with $d_p>0(p=1,2,\ldots,n)$ is the self-feedback connection weight matrix, $A(t)=(a_{pq})_{n\times n}\in\mathbb{C}^{n\times n}$ and $B(t)=(b_{pq})_{n\times n}\in\mathbb{C}^{n\times n}$ are, respectively, the connection weight matrix without and with time delays, $f(z)=((f_1(z_1),f_2(z_2),\ldots,f_n(z_n)))^T:\mathbb{C}^n\to\mathbb{C}^n$ is the activation function of the neurons, $H(t)=((H_1(t),H_2(t),\ldots,H_n(t)))^T\in\mathbb{C}^n$ is the external input vector-valued function. $\tau(t)\geq 0$ correspond to the transmission delays. $\Delta z(t_k)=z(t_k^+)-z(t_k^-)$ are the impulses at moments t_k and $0<t_1<t_2<\cdots$ is strictly increasing sequence such that $\lim_{k\to\infty}t_k=+\infty$, $\{t_k\}$ are called points of jump.

For the sake of convenience, we introduce the following notations: $\widehat{a}_{pq}=\max_{t\in[0,\omega]}|a_{pq}|$, $\tilde{a}_{pq}=\min_{t\in[0,\omega]}|a_{pq}|$, $\widehat{b}_{pq}=\max_{t\in[0,\omega]}|b_{pq}|$, $\tilde{b}_{pq}=\min_{t\in[0,\omega]}|b_{pq}|$, $\widehat{d}_{pq}=\max_{t\in[0,\omega]}|d_{pq}|$, $\tilde{d}_{pq}=\min_{t\in[0,\omega]}|d_{pq}|$.

Throughout this paper, we have the following assumptions:

(H1) For $q=1,2,\ldots,n$, there exist positive constants M_q such that $|f_q(z)|\leq M_q$.

(H2) All $d_p(t)$ are positive continuous periodic real-functions with period ω; $a_{pq}(t)$, $b_{pq}(t)$, $H_p(t)$ are all continuous periodic functions with period ω. i.e., there exists a constant $\omega>0$ such that $d_p(t+\omega)=d_p(t)$, $a_{pq}(t+\omega)=a_{pq}(t)$, $b_{pq}(t+\omega)=b_{pq}(t)$, $H_p(t+\omega)=H_p(t)(p=1,2,\ldots,n)$ for all $t>0$, $p=1,2,\ldots,n$.

(H3) There exists a positive integer $m>0$ such that $t_{k+m}=t_k+\omega$, $J_{p(k+m)}=J_{pk}$, $p=1,2,\ldots,n$; $k\in\mathbb{Z}^+$.

By separating the state, the connection weight, the activation function and the external input into its real and imaginary part,

Download English Version:

https://daneshyari.com/en/article/494482

Download Persian Version:

https://daneshyari.com/article/494482

Daneshyari.com