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Impulsive control for the synchronization of coupled neural networks with reaction–diffusion terms

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ABSTRACT

The impulsive control method is utilized to achieve the synchronization of coupled reaction–diffusion neural networks with time-varying delay. By combining the Lyapunov functional method with the impulsive delay differential inequality and comparison principle, a few sufficient conditions are derived to guarantee the global exponential synchronization of coupled neural networks with reaction–diffusion terms. Especially, the estimate for the exponential convergence rate is also given, which relies on time delay, system parameters and impulsive interval. Finally, numerical examples are provided to demonstrate the correctness and effectiveness of our results.

1. Introduction

As is well known, complex networks extensively exist in diverse areas of real-life world, such as food webs, biological neural networks, social networks, genetic regulatory networks, electrical power grids, cellular networks, metabolic systems, World Wide Web, and so on. Due to their broad applications, they have been receiving a great deal of attention in the investigation of topology and dynamical behavior for various complex networks. Specifically, widespread attention has been centered on synchronization problem about complex dynamical networks. Up to now, a wide variety of interesting results on synchronization have been derived for complex networks [1–8]. Cheng et al. [2] studied the adaptive pinning synchronization of delayed complex networks with nonlinear coupling by employing Lyapunov stability theory. In [4], several criteria were derived to guarantee the synchronization of complex network model with fractional order chaotic nodes by using the LaSalle invariance principle.

Up to date, many researchers have devoted much effort to synchronization problem for arrays of coupled neural networks (CNNs) because of its wide applications in different fields. For instance, the linear coupled cellular neural networks have been triumphantly applied to a secure communication system [9] and the electronic circuits [10]. In [11], the authors presented an architecture of CNNs to memorize and reproduce complex oscillatory patterns as synchronization states. Moreover, the research about synchronization of CNNs is a significant step to comprehend brain science [12]. Therefore, studying synchronization problem about coupled neural networks has both practical and theoretical significance [13–23]. Yang et al. [14] studied the finite-time synchronization for an array of coupled neural networks with discontinuous activation functions and mixed delays. A few sufficient conditions were gained to ensure finite-time synchronization of the networks by designing suitable controller. In [15], the authors investigated the exponential synchronization problem for coupled fuzzy neural networks with mixed time-delays and disturbances by utilizing some stochastic analysis methods. Song et al. [17] analyzed the pinning synchronization of coupled delayed neural networks with both constant and delayed couplings.

Nevertheless, in a lot of research works on coupled neural networks [13–23], the diffusion effects are not yet taken into account. Actually, the diffusion phenomena inevitably appear in electric circuits and neural networks once electrons transport in a nonuniform electromagnetic field [24,25]. Hence, it is extremely important to investigate the synchronization in coupled neural networks with reaction–diffusion terms [26–31]. Liu et al. [26] analyzed the μ-synchronization and pinning control problems for coupled reaction–diffusion neural networks (CRDNNs) with unbounded time-delays and Dirichlet boundary conditions. In [28], a sufficient condition ensuring synchronization was derived by utilizing the correlation between output strict passivity and

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synchronization. Wang et al. [30] considered the adaptive synchronization for an array of linearly CRDNNs with time delays.

Impulsive control technique has been widely applied to realize stabilization and synchronization for nonlinear unstable dynamical systems and chaotic systems. The major idea about impulsive control is to change the states of continuous dynamic systems via discontinuous control inputs at certain time instants. From the control point of view, impulsive control method is effective and robust in the research of stability analysis, since it needs only small control gains. Moreover, utilizing the impulsive control method is very advantageous in practical applications due to reduced control cost [33]. More recently, a great deal of attention has been given to impulsive control problem of neural networks with reaction–diffusion terms [32–34]. Hu et al. [32] gave a few sufficient conditions dependent on the diffusion coefficients to guarantee the global exponential stability and synchronization of delayed reaction–diffusion neural networks using the impulsive control strategy. In [33], the authors considered the global exponential stability for Cohen–Grossberg neural networks with reaction–diffusion terms and Dirichlet boundary conditions via the impulsive control method. Yang et al. [34] discussed the problem of stochastic synchronization for reaction–diffusion neural networks under impulsive controller with mixed delays. Several sufficient criteria were established by means of the impulsive differential inequality and the properties of random variables. Obviously, it is also beneficial to apply the impulsive control method to analyze synchronization problem for CRDNNs. Unfortunately, there are very few results concerning the impulsive control for synchronization of coupled reaction–diffusion neural networks. In [35], the authors studied global exponential synchronization for linearly CRDNNs with time-varying delays by adding impulsive controller to a small fraction of nodes.

The main contributions of this paper are as follows. First, one delay-independent global exponential synchronization condition is derived by using impulsive delay differential inequality. Second, two delay-dependent global exponential synchronization criteria are established by means of suitable Lyapunov functionals and in terms of several linear matrix inequalities. Third, the estimate for exponential convergence rate is provided, which relies on time delay, system parameters and impulsive interval.

The structure of this paper is organized as follows. In Section 2, we introduce some necessary notations and a lemma, which will be utilized throughout this paper. The major results of this paper are obtained in Section 3. In Section 4, numerical examples are given to show the effectiveness of the proposed results. We ultimately draw our conclusions in Section 5.

2. Preliminaries

2.1. Notations

$\Omega=\{x=(x_1,x_2,\dots,x_q)^T \mid |x_k|<l_k, k=1,2,\dots,q\}$ is an open bounded domain in $\mathbb{R}^q$ with smooth boundary $\partial\Omega$. $\mathrm{PC}[[-\tau,+\infty),\mathbb{R}]:=\{\phi:[-\tau,+\infty)\to\mathbb{R}, \phi(t)$ is continuous everywhere except for the points $t_k, k\in\mathbb{N}$ at which $\phi(t_k^+)=\phi(t_k)$ and $\phi(t_k^-)$ exist$\}$. The fixed moments t_k satisfy $0=t_0<t_1<t_2<\cdots<t_k<\cdots$ and $\lim_{k\to+\infty}t_k=+\infty, k\in\mathbb{N}$, let $T_{k-1}=t_k-t_{k-1}, T_{\min}=\inf_{k\in\mathbb{N}}\{T_{k-1}\}$, $T_{\max}=\sup_{k\in\mathbb{N}}\{T_{k-1}\}$. For any $e(x,t)=(e_1(x,t),e_2(x,t),\dots,e_n(x,t))^T \in\mathbb{R}^n, (x,t)\in\Omega\times\mathbb{R}$, $\|e(\cdot,t)\|_2$ represents

$$\|e(\cdot,t)\|_2=\left(\int_\Omega\sum_{i=1}^n e_i^2(x,t)dx\right)^{1/2}.$$

For any $\Psi(x,t)=(\psi_1(x,t),\psi_2(x,t),\cdots,\psi_n(x,t))^T\in\mathbb{R}^n, (x,t)\in\Omega\times[-\tau,+\infty)$, we can define

$$\|\Psi(\cdot,t)\|_\tau=\sup_{-\tau\le\theta\le0}\|\Psi(\cdot,t+\theta)\|_2$$

for $t\in[0,+\infty)$.

2.2. Lemma

Lemma 2.1. *(see [33]) Let $0\le\tau_i(t)\le\tau_i, \sigma>0, m_i\ge0, m\in\mathbb{R}$, $i=1,2,\dots,n$. Assume that $m+\frac{\ln\sigma}{\rho}+\sigma^{\mathrm{sgn}(\ln\sigma)}\sum_{i=1}^n m_i<0$ and $u(t)\in\mathrm{PC}[[-\tau,+\infty),\mathbb{R}^+]$ satisfies*

$$\begin{cases} D^+u(t)\le mu(t)+\sum_{i=1}^n m_iu(t-\tau_i(t)), & t\ge0,\\ u(t_k)\le\sigma u(t_k^-), & k\in\mathbb{N},\\ u(t)=\phi(t), & -\tau\le t\le0,\end{cases}$$

in which $\tau=\max_{i=1,2,\dots,n}\{\tau_i\}$, and $\phi(t)$ is bounded and continuous on $[-\tau,0]$, then

$$u(t)\le\sigma^{\mathrm{sgn}(\ln\sigma)}e^{-\lambda t}\sup_{-\tau\le s\le0}\phi(s),\quad t\ge0,$$

where $\lambda>0$ is a unique solution of

$$\lambda+m+\frac{\ln\sigma}{\rho}+\sigma^{\mathrm{sgn}(\ln\sigma)}\sum_{i=1}^n m_ie^{\lambda\tau_i}=0,$$

in which $\rho\ge T_{\max}$ if $\sigma<1$, otherwise, $\rho\le T_{\min}$;

$$\mathrm{sgn}(\ln\sigma)=\begin{cases}-1 & \text{if}\quad \sigma<1,\\ 0 & \text{if}\quad \sigma=1,\\ 1 & \text{if}\quad \sigma>1.\end{cases}$$

Remark 1. In this paper, we always assume $0<T_{\min}\le T_{\max}<+\infty$. Lemma 2.1 is very significant for us to analyze the global exponential synchronization of CRDNNs via impulsive control.

3. Main results

Consider a single reaction–diffusion neural network with Dirichlet boundary conditions which is described as follows:

$$\frac{\partial w_i(x,t)}{\partial t}=d_i\Delta w_i(x,t)-a_iw_i(x,t)+J_i+\sum_{j=1}^n b_{ij}f_j(w_j(x,t)), \tag{1}$$

where $i=1,2,\dots,n$, n is the number of neurons in the network; $x=(x_1,x_2,\dots,x_q)^T\in\Omega\subset\mathbb{R}^q$; $w_i(x,t)\in\mathbb{R}$ is the state of the ith neuron at time t and in space x; $\Delta=\sum_{k=1}^q\frac{\partial^2}{\partial x_k^2}$ is the Laplace diffusion operator on Ω; $d_i>0$ represents the transmission diffusion coefficient along the ith neuron; $a_i>0$ denotes the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external input; J_i is a constant external input; $f_j(\cdot)$ represents the activation function of the jth neuron; b_{ij} represents the strength of the jth neuron on the ith neuron.

The initial value and boundary value conditions of system (1) are given in the following form:

$$w_i(x,0)=\phi_i(x), x\in\Omega, \tag{2}$$

$$w_i(x,t)=0, (x,t)\in\partial\Omega\times[0,+\infty), \tag{3}$$

where $\phi_i(x)(i=1,2,\dots,n)$ is continuous and bounded on Ω.

In the paper, the function $f_j(\cdot)(j=1,2,\dots,n)$ satisfies the Lipschitz condition, and there exists positive constant ρ_j such that

$$|f_j(\xi_1)-f_j(\xi_2)|\le\rho_j|\xi_1-\xi_2|$$

for any $\xi_1,\xi_2\in\mathbb{R}$, where $|\cdot|$ is the Euclidean norm.

We can rewrite system (1) in a compact form

$$\frac{\partial w(x,t)}{\partial t}=D\Delta w(x,t)-Aw(x,t)+J+Bf(w(x,t)), \tag{4}$$